FDPLL – A First-Order Davis-Putnam-Logemann-Loveland Procedure

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Theorem Proving

Theorem proving is about . . .

- Logics (Propositional, First-Order, Higher-Order, Modal, Description, . . .)
- Calculi and proof procedures (Resolution,. . .)
- Systems (Interactive, Automated)
- Applications (Knowledge Representation, Verification, . . .)

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Milestones

60s: Calculi: DPLL, Resolution, Model Elimination

70s: Logic Programming

80s: Knowledge Representation

90s: "A Basis for Applications"

2000s: Semantic Web, Ontologies, SW-Engineering

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Two Separated Worlds

	First-Order Reasoning	**Propositional Reasoning**
Techniques	Resolution	DPLL
	Model Elimination	OBDD
	Hyper Linking	Stalmarck's Method
		Tableaux
		Stochastic (GSAT)
Systems	E, Otter, Setheo, SNARK, Spass, Vampire	Chaff, SMV, Heerhugo, FACT, WalkSat
Applications	SW-Verification (Limited)	Symbolic Model Checking
	Mathematics	Mathematics
	Discourse Representation	Planning, Description Logics
	TPTP	Nonmonotonic Reasoning

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Can couple these worlds more closely?

Motivation

DPLL: Successfully used for propositional logic

FDPLL: New lifting of DPLL to **f**irst-order logic

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- Use successful first-order techniques (unification, redundancy tests)

Unification:

$$\mathrm{unify}\{P(a,y),\ P(x,f(x))\}$$
$$= \{P(a,f(a))\}$$

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$$\text{Axioms} \overset{?}{\models} \text{Conjecture}$$

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- (Dream) Bring first-order reasoning to domains that are successfully tackled with propositional DPLL

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Overview

Propositional DPLL as a semantic tree method

First-Order DPLL so far

FDPLL

Relation to other calculi

Notation

Propositional clause: a disjunction of literals, e.g.

$$A \lor B \lor \neg C \lor \neg D$$

Propositional clause set: a finite set of propositional clauses.

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$$A \vee B \vee \neg C \vee \neg D$$

Propositional clause set: a finite set of propositional clauses.

Interpretation: maps atoms to $\{\text{true}, \text{false}\}$, e.g.

A	B	C	D
true	false	true	false

Representation by consistent sets of literals, e.g. (all the same)

$$\{A, C\} \qquad \{A, \neg B, C\} \qquad \{A, \neg B, C, \neg D\}$$

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Model: an interpretation such that every clause is satisfied, e.g.

$$\{A, C\} \models \{A \lor B \lor \neg C \lor \neg D\}$$

$$\{A, C\} \not\models \{A \lor B \lor \neg C \lor \neg D,\ \neg A \lor B\}$$

A clause set is satisfiable iff a model for it exists, otherwise unsatisfiable.

Propositional DPLL as a Semantic Tree Method

(1) $A \vee B$ (2) $C \vee \neg A$ (3) $D \vee \neg C \vee \neg A$ (4) $\neg D \vee \neg B$

$\langle$empty tree$\rangle$

$\{\} \not\models A \vee B$

$\{\} \models C \vee \neg A$

$\{\} \models D \vee \neg C \vee \neg A$

$\{\} \models \neg D \vee \neg B$

- A Branch stands for an interpretation
- Purpose of splitting: Satisfy a clause that is currently "false"
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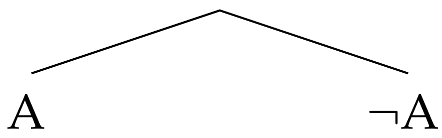

$\{A\} \models A \vee B$

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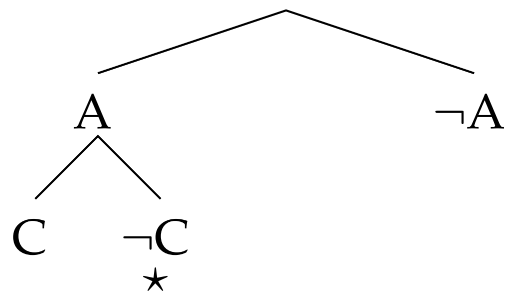

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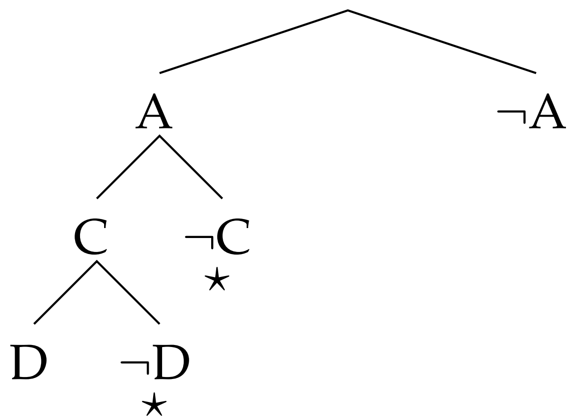

$\{A, C, D\} \models A \vee B$

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Model $\{A, C, D\}$ found.

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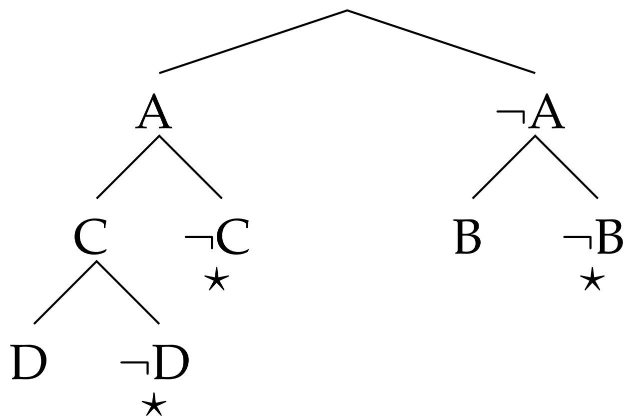

$\{B\} \models A \vee B$

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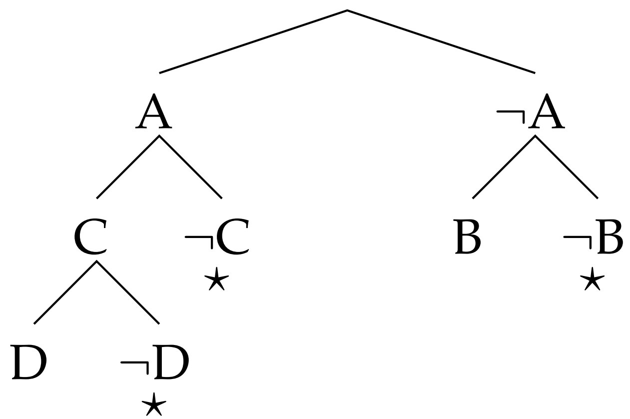

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- **Sound and complete, also for (minimal) model reasoning**

DP vs. DPLL

Two versions of the main inference rule:

Davis, Putnam 1960: "Rule for eliminating atomic formulas":

1. Select an atom A
2. Resolve (!) on all clauses $\mathrm{A} \vee \ldots$ and $\neg\mathrm{A} \vee \ldots$
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Davis 1963; Chinlund, Davis, Hinman, McIlroy 1964:

Improvement of first-order case.

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Relation to other calculi

DPLL - The First-Order Case (1962)

Preprocessing:

Given Formula		Clause Form
$\forall x\ \exists y\ P(y,x)$ $\rightarrow \exists y\ P(y,a)$	$\longrightarrow$	$P(f(x),x)$ $\neg P(y,a)$

Outer loop:
Grounding

Inner loop:
Propositional
DPLL

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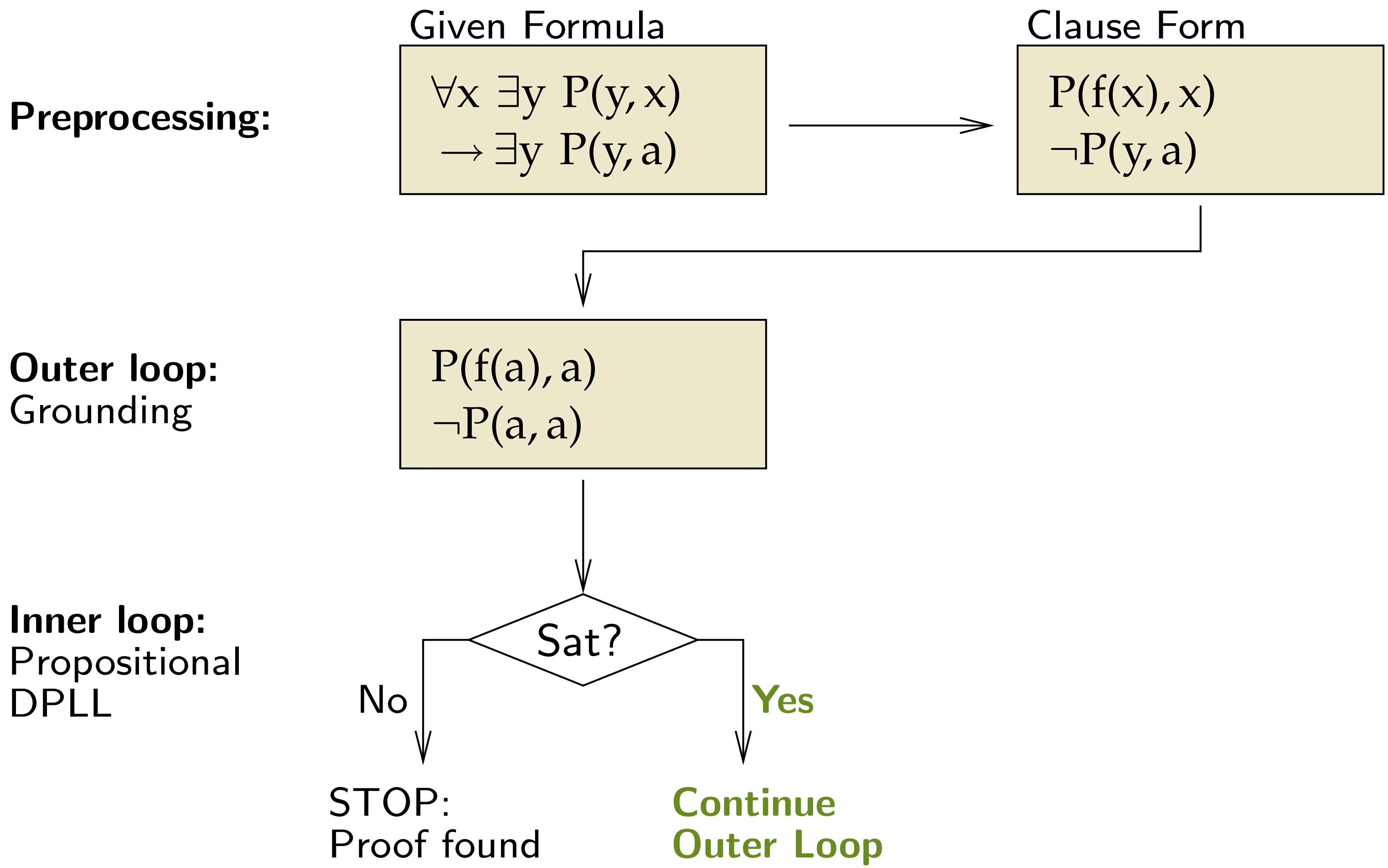

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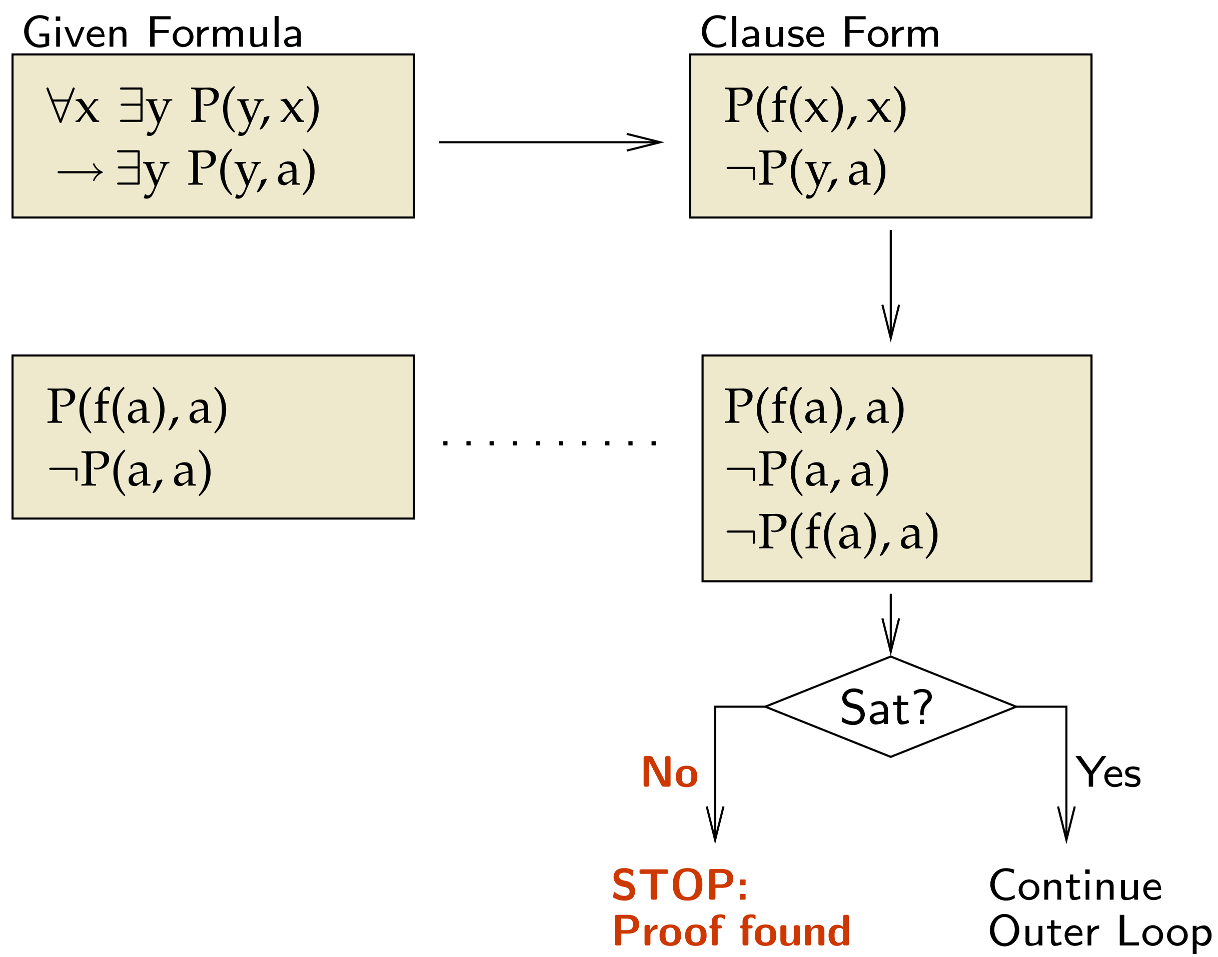

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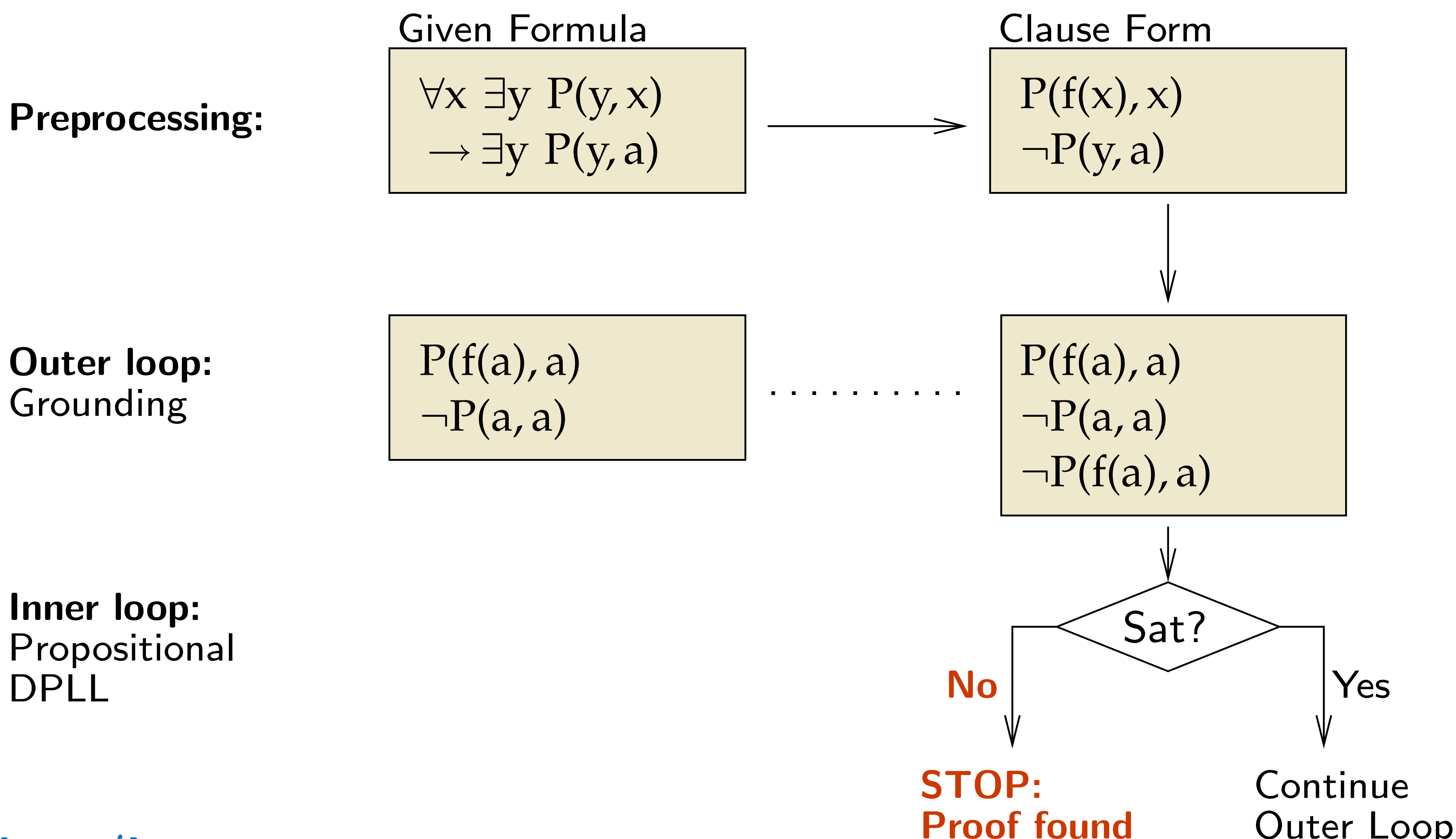

Problems/Issues:

- Controlling the grounding process in outer loop (irrelevant clauses)
- Repeat work across inner loops
- Weak redundancy criterion within inner loop

Controlling the Grounding Process

Davis 1963; Chinlund, Davis, Hinman, McIlroy 1964:

"Linked Conjunct Method":

Admissible clause set:	Non-admissible clause set:
$P(a) \lor Q(a)$	$P(b) \lor Q(a)$
$\neg P(a) \lor Q(a)$	$\neg P(a) \lor Q(a)$
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Every literal has a mate	The literal $P(b)$ is pure

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Some more recent work in this tradition:

Lee&Plaisted 1992, Chu&Plaisted 1994, Plaisted & Zhu 1997: (O)(S)HL

Billon 1996: Disconnection Method

Baumgartner 1998: Hyper Tableaux Next Generation

Parkes 1999: Lifted Search Engines for Satisfiability

May show very good performance!

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- Instance based methods reduce first-order to propositional logic

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- E.g. Resolution performs intrinsic first-order reasoning

  Advantages:

  Representation: Infinitely many inferences finitely represented:

$$\frac{P(f(x),x) \qquad \neg P(y,z) \lor Q(y,z)}{Q(f(x),x)}$$

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  Infinitely many inferences in instance based methods

  Redundancy testing: E.g. by subsumption:

$$\neg P(y,z) \quad \text{subsumes} \quad \neg P(y,y) \lor Q(y,y)$$

  Lack of redundancy testing in instance based methods

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Propositional DPLL as a semantic tree method ✔

First-Order DPLL so far ✔

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Meta-Level Strategy

Lifted data structures:

	Propositional Reasoning	First-Order Reasoning
Resolution	$A \vee \neg B \vee C$	$P(x, y) \vee \neg Q(x, z) \vee R(y, z)$

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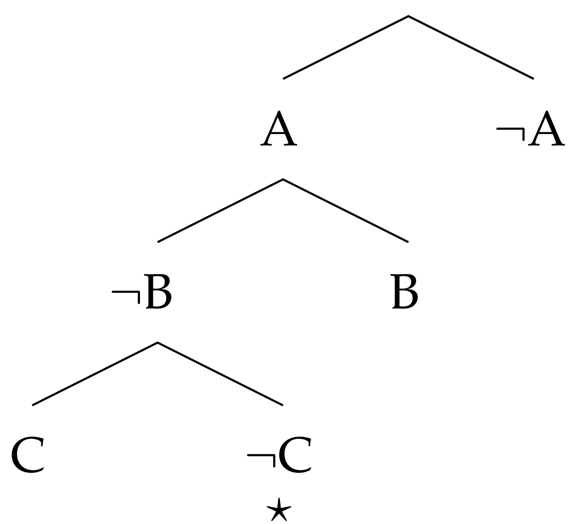
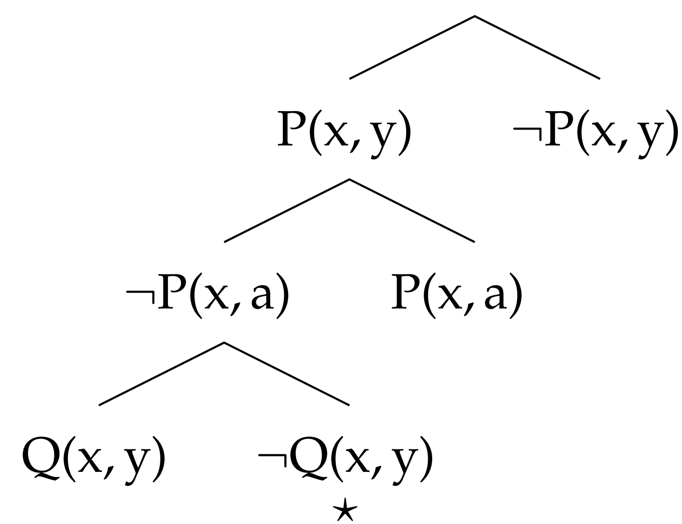
DPLL		

FDPLL: First-Order Semantic Trees

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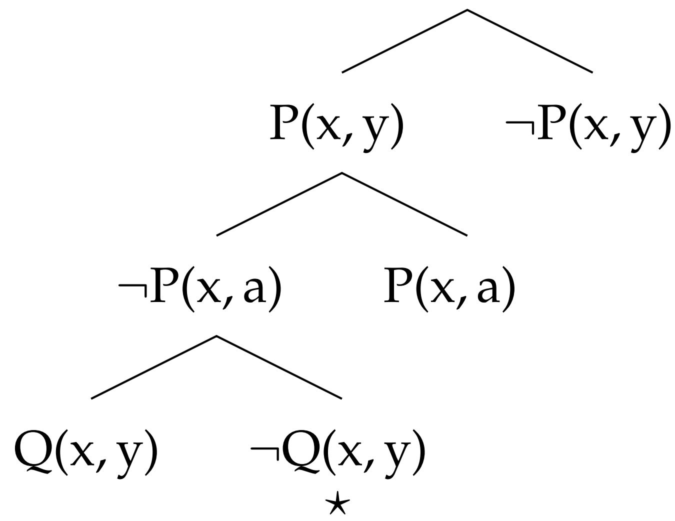

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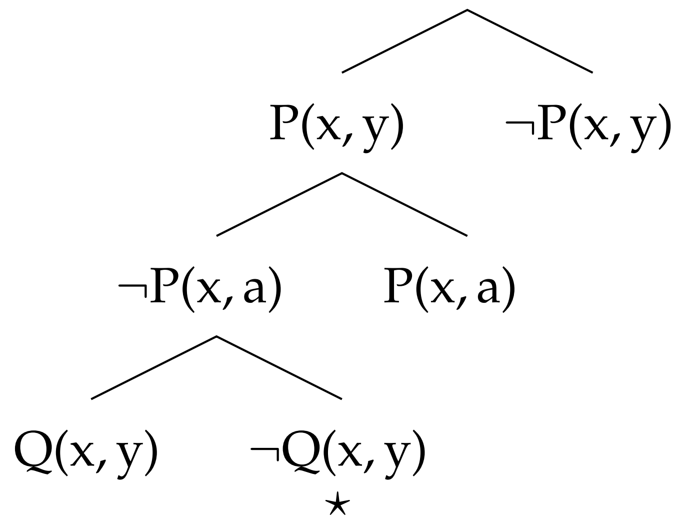

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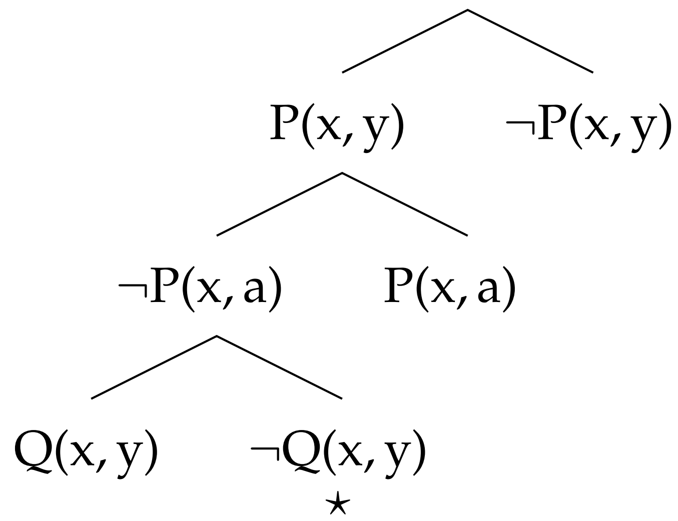

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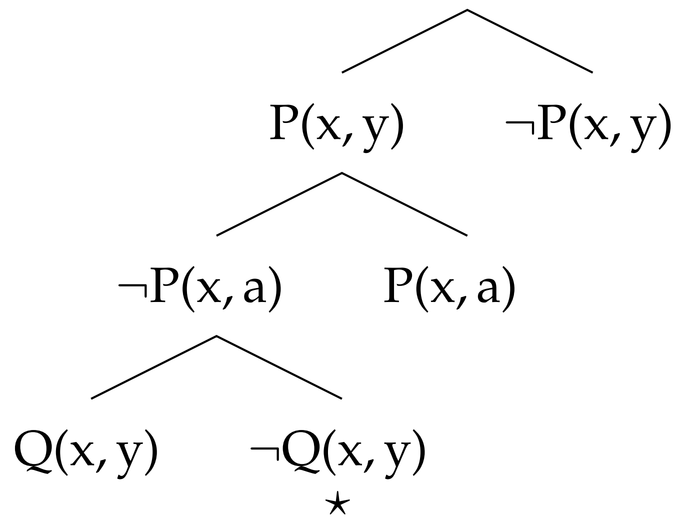

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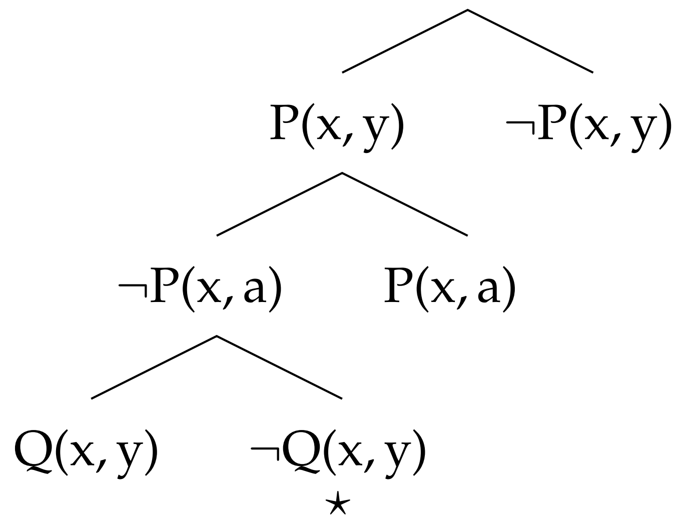

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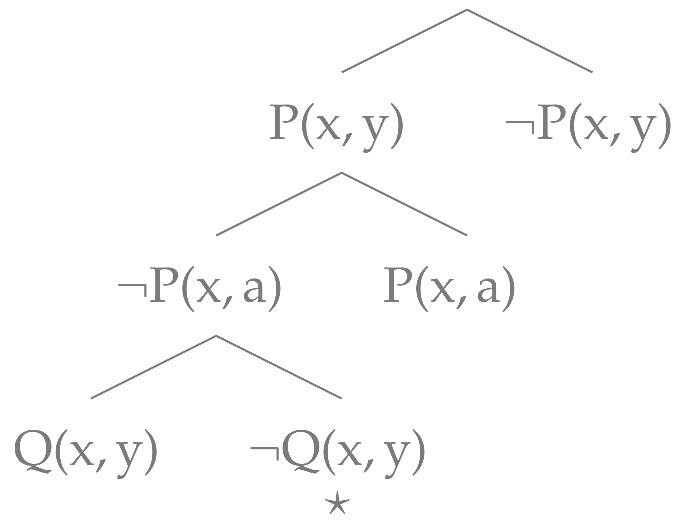

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Extracting an Interpretation from a Branch

Branch $\mathcal{B}$:

$$\Big| \\ P(x, y)$$

Interpretation $[\![\mathcal{B}]\!] = \{\ldots\}$:

- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying opposite truth values.

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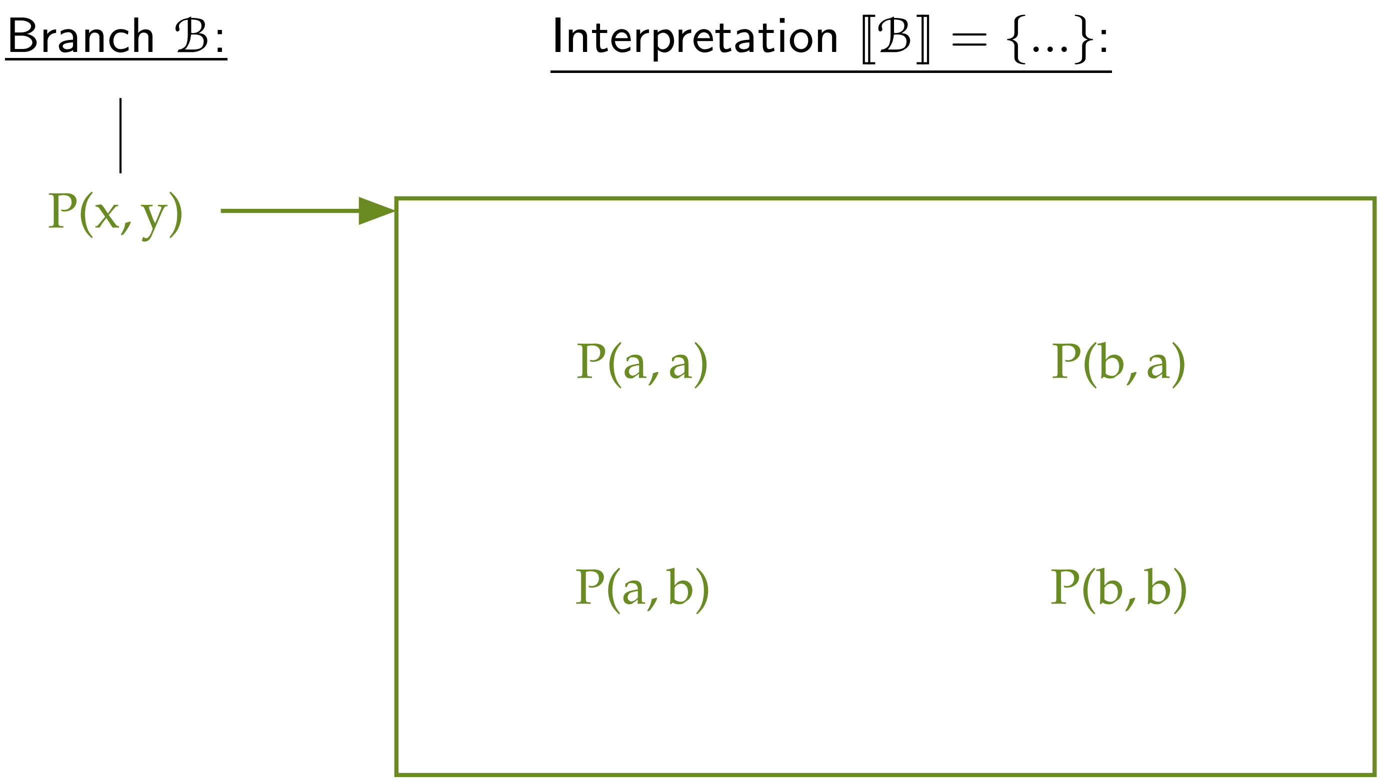

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Branch $\mathcal{B}$:

$P(x, y)$

$\neg P(a, y)$

Interpretation $[\![\mathcal{B}]\!] = \{\ldots\}$:

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$P(a, b)$	$P(b, b)$

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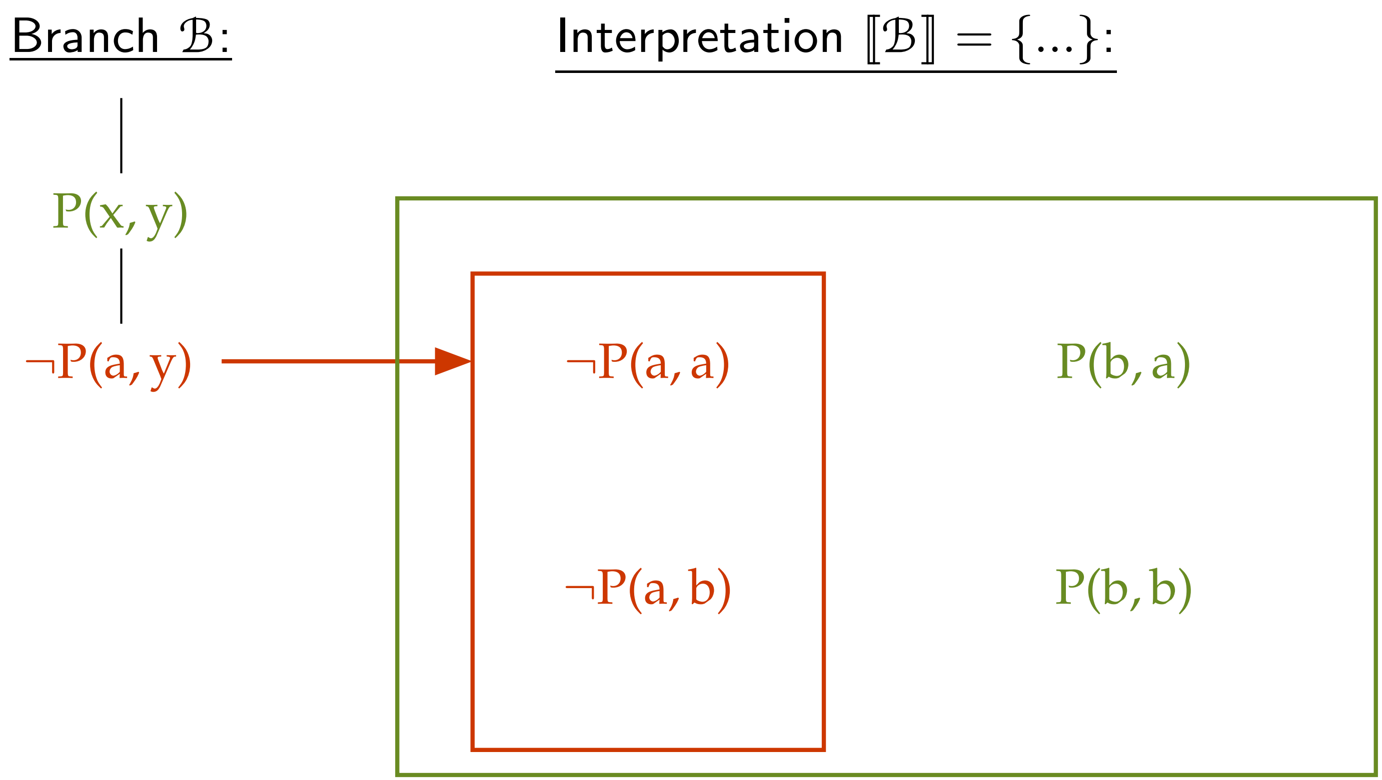

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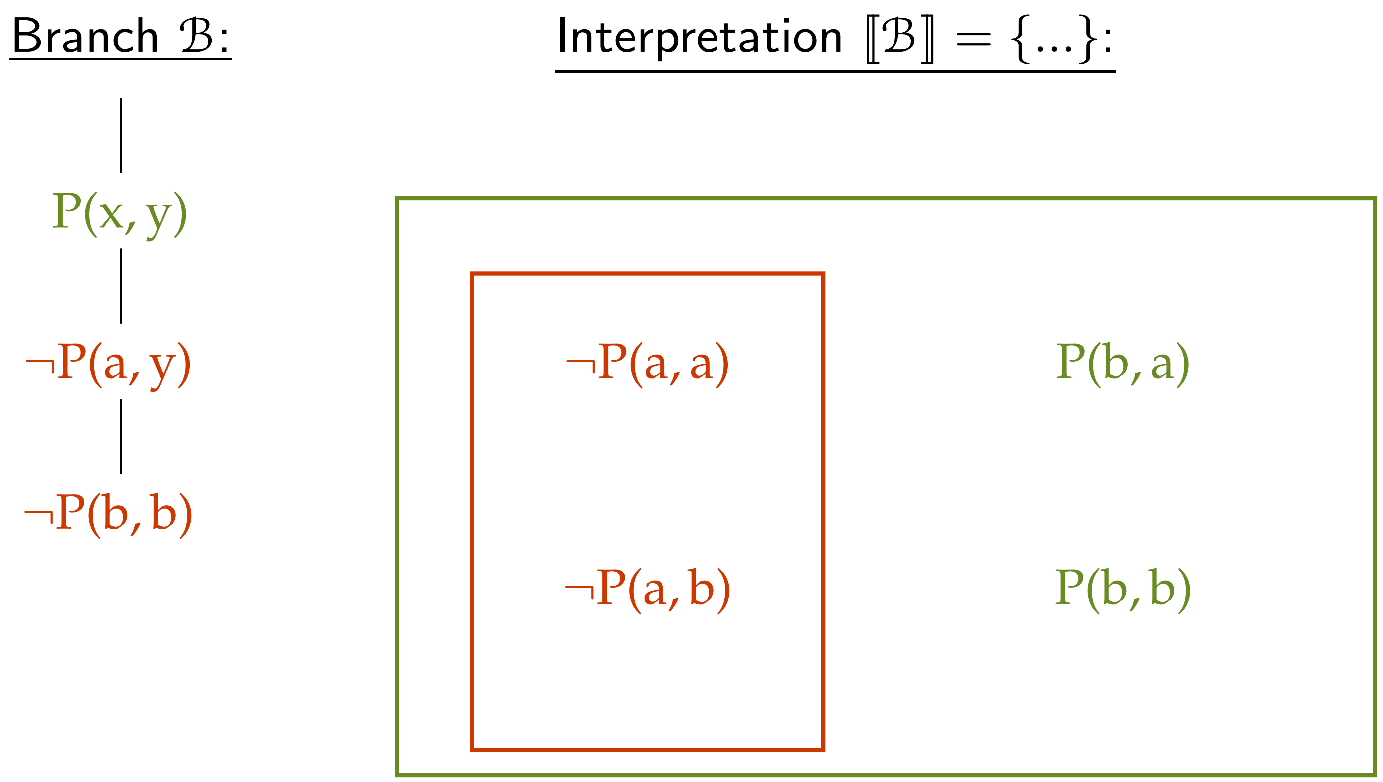

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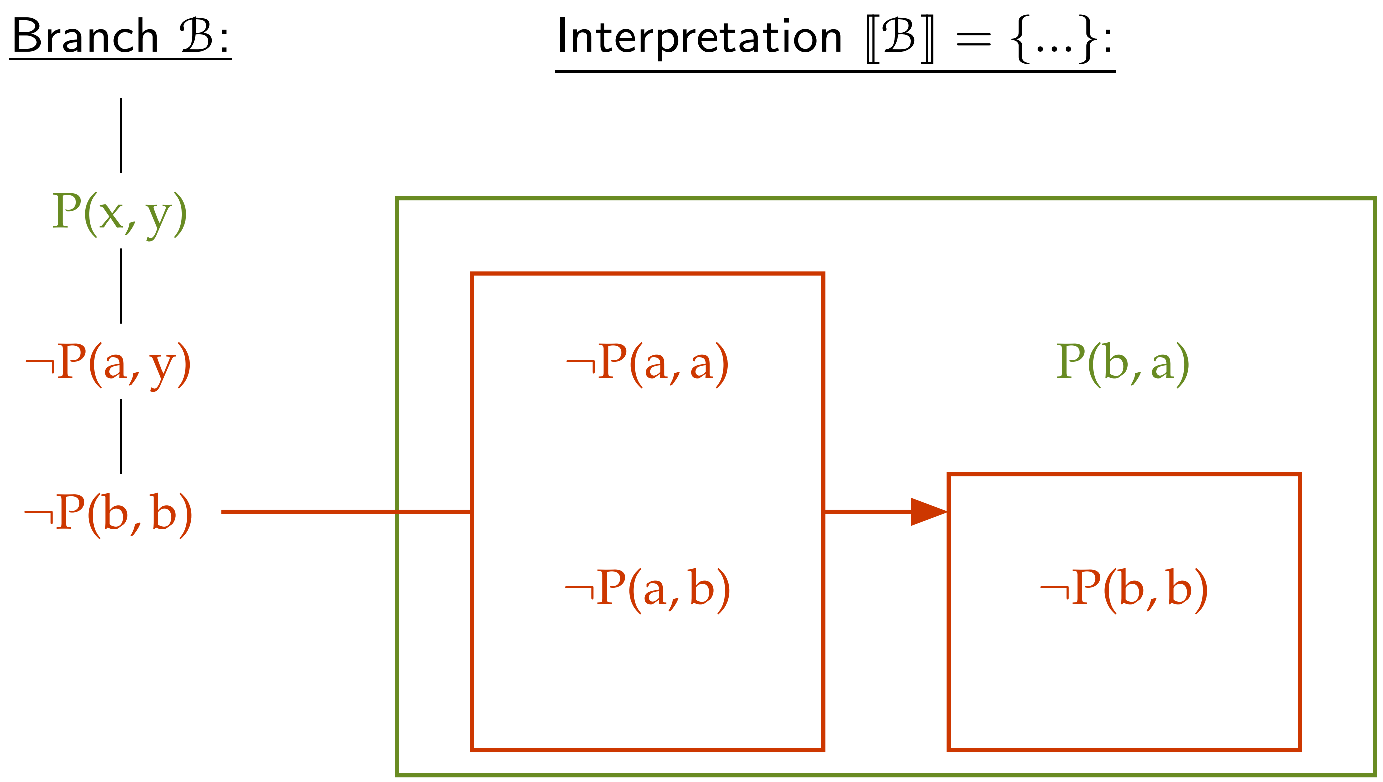

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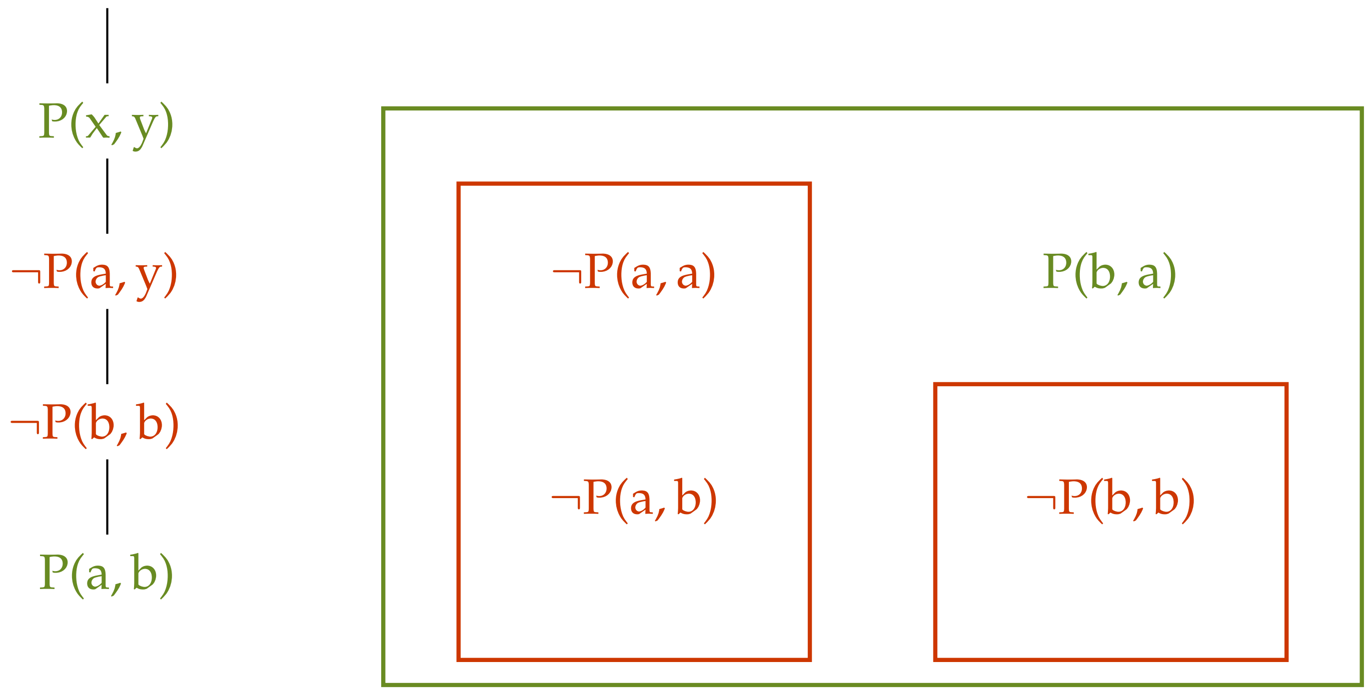
Extracting an Interpretation from a Branch

Branch $\mathcal{B}$:

Interpretation $[\![\mathcal{B}]\!] = \{\ldots\}$:



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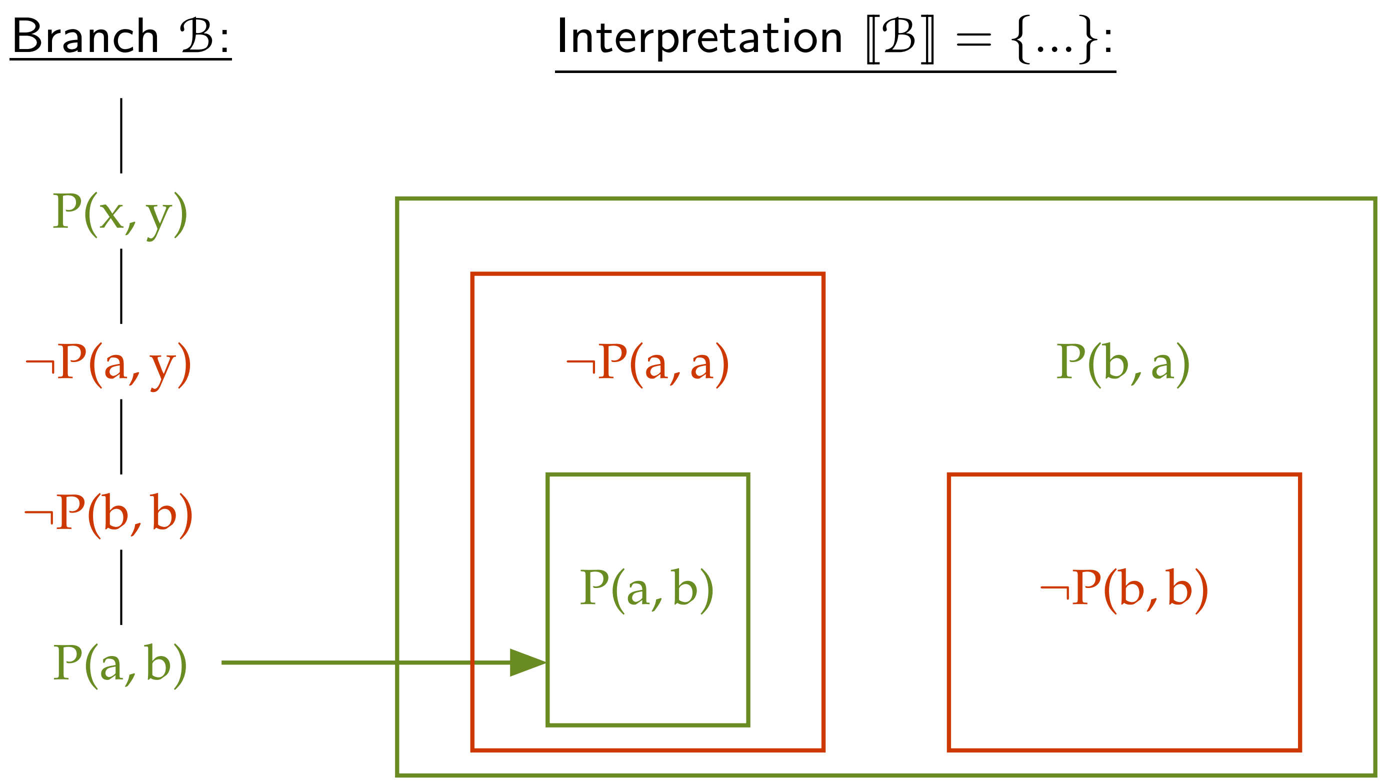
Extracting an Interpretation from a Branch



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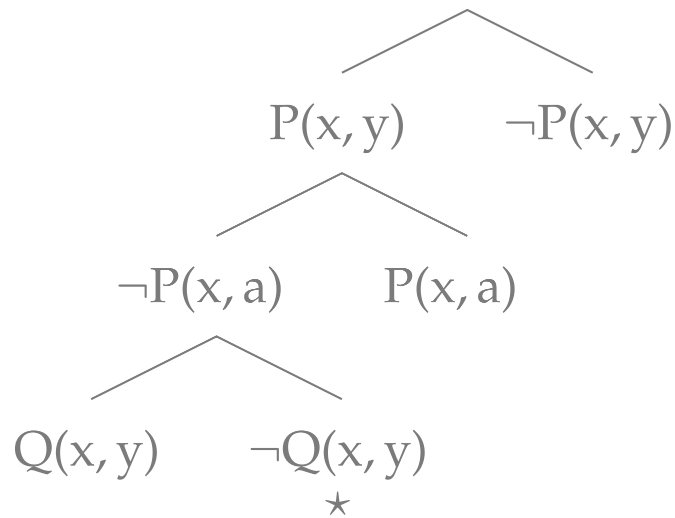
Extracting an Interpretation from a Branch

Branch $\mathcal{B}$:	Interpretation $[\![\mathcal{B}]\!] = \{\ldots\}$:
$P(x,y)$	
$\neg P(a,y)$	$\{\ \neg P(a,a),\quad P(b,a),$
$\neg P(b,b)$	
	$P(a,b),\quad \neg P(b,b)\ \}$
$P(a,b)$	

- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying opposite truth values.
- The order of literals does not matter.

First-Order Semantic Trees



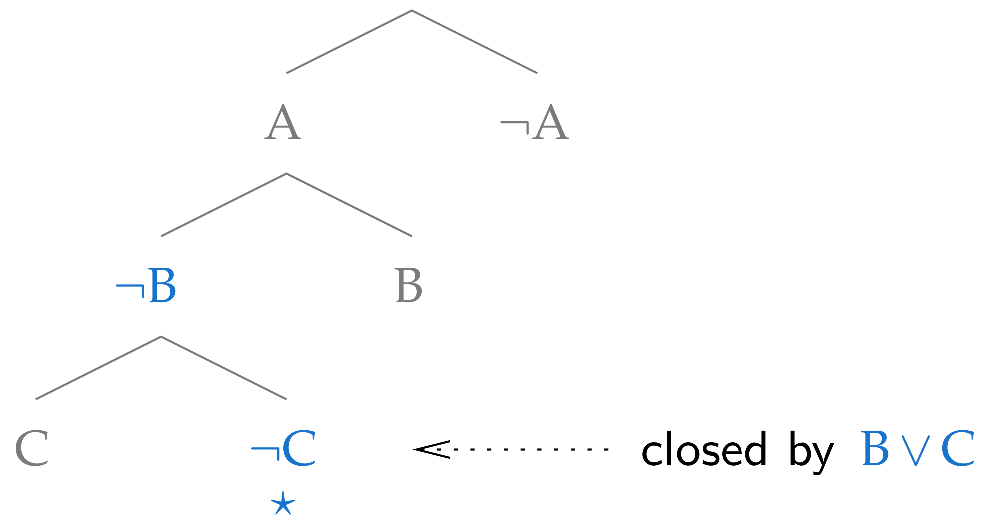
Issues:

- One-branch-at-a-time approach desired
- How are variables treated?
  (a) Universal, as in Resolution?, (b) Rigid, as in Tableaux? (c) Schema!
- How to extract an interpretation from a branch? ✔
- **When is a branch closed?**
- How to construct such trees (calculus)?

Calculus: Branch Closure

Purpose: Determine if branch elementary contradicts an input clause.

Propositional case:



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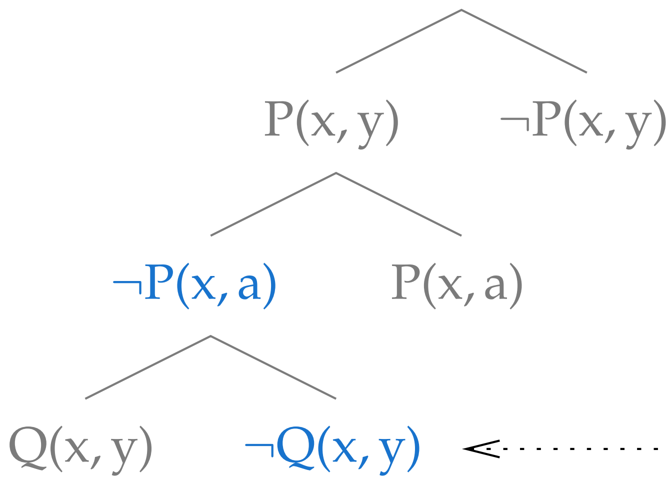
FDPLL case:



closed by $P(x,y) \vee Q(x,x)$?

Calculus: Branch Closure

Purpose: Determine if branch elementary contradicts an input clause.

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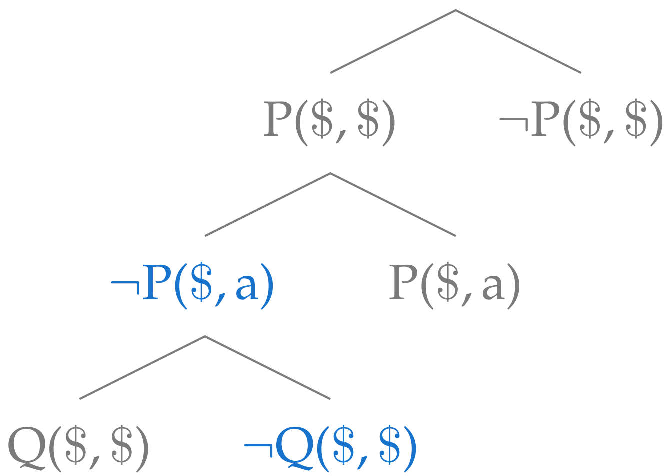


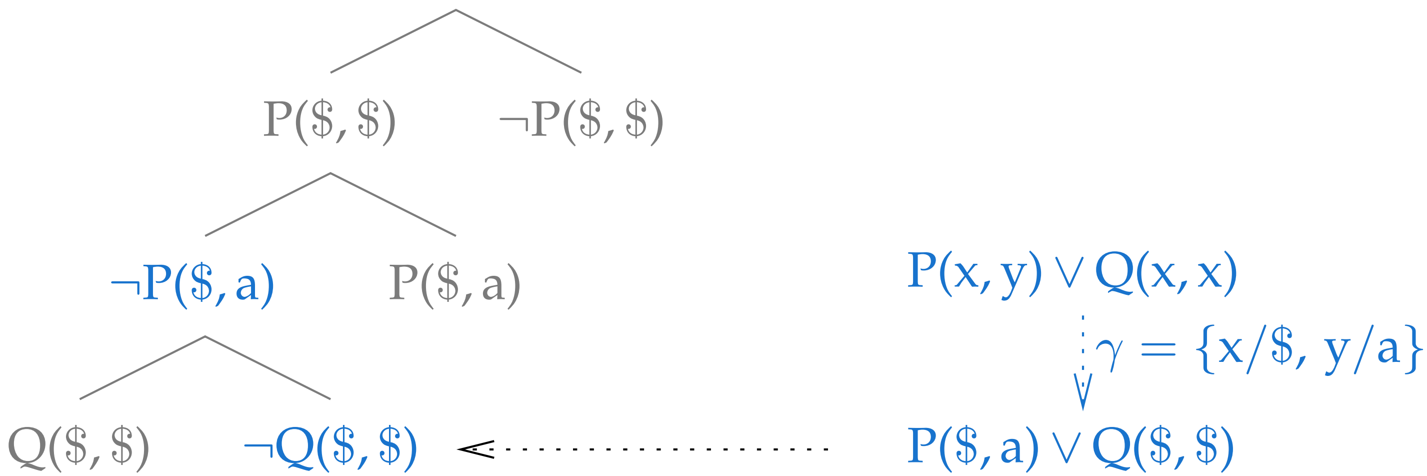
$P(x, y) \lor Q(x, x)$

1. Replace all variables in tree by a constant \$. Gives propositional tree
2.
3.

Calculus: Branch Closure

Purpose: Determine if branch elementary contradicts an input clause.

FDPLL case:



1. Replace all variables in tree by a constant $\$$. Gives propositional tree
2. Compute matcher γ to propositionally close branch
3.

Calculus: Branch Closure

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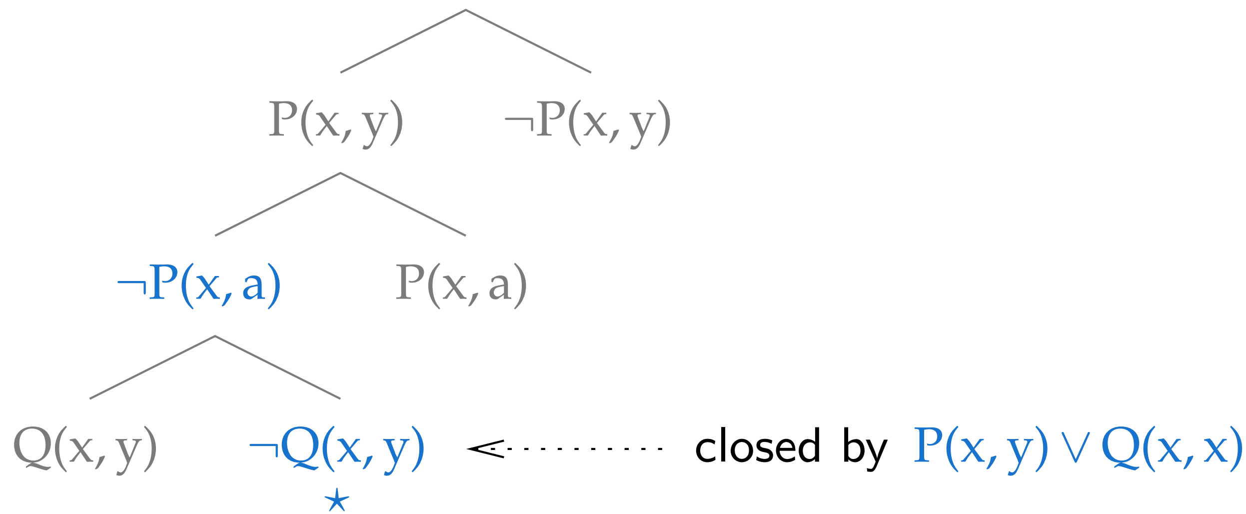
FDPLL case:



1. Replace all variables in tree by a constant $\$$. Gives propositional tree
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3. Mark branch as closed $(\star)$

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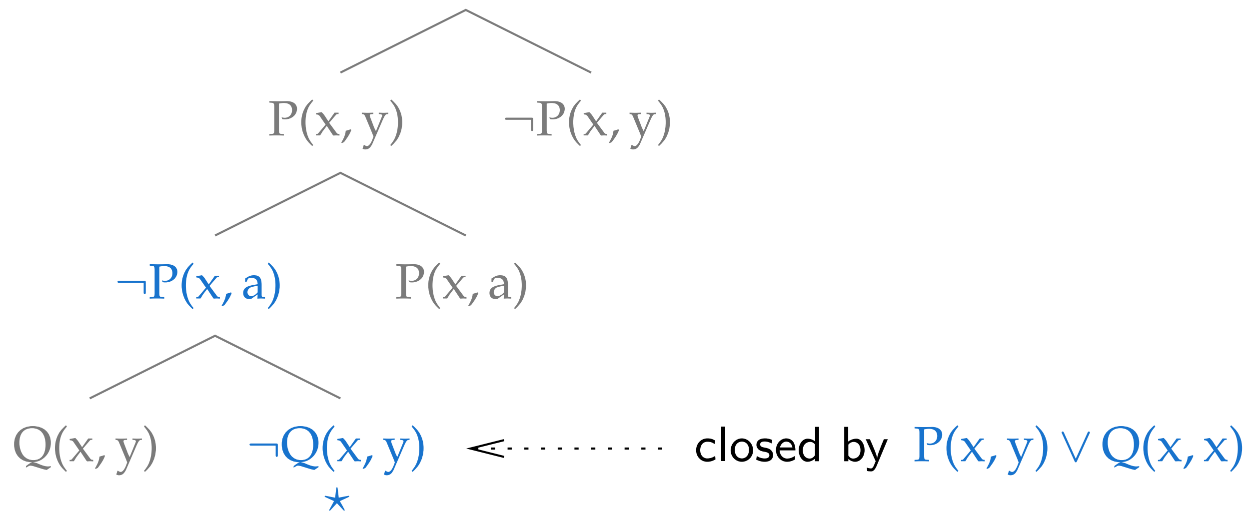

1. Replace all variables in tree by a constant $\$$. Gives propositional tree
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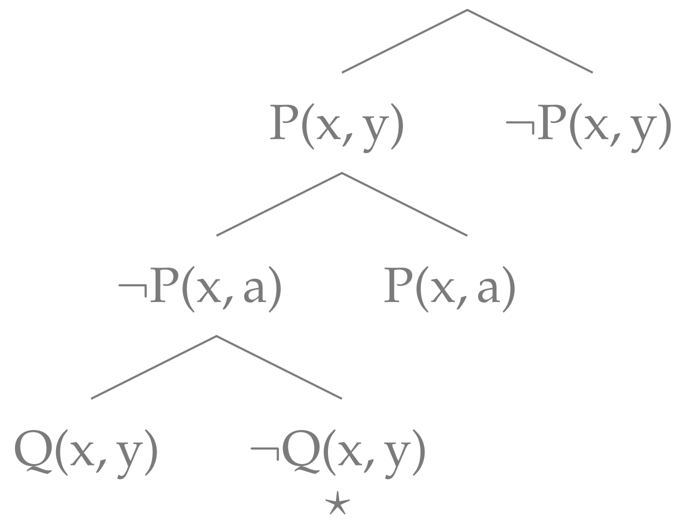
Theorem: FDPLL is sound (because propositional DPLL is sound), and splitting can be done with arbitrary literal.

First-Order Semantic Trees



Issues:

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  (a) Universal, as in Resolution?, (b) Rigid, as in Tableaux? (c) Schema!
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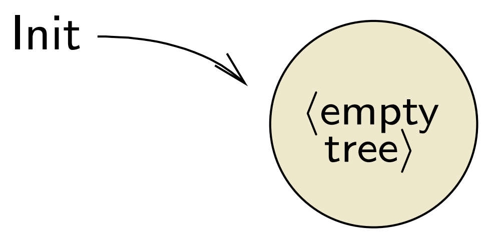
FDPLL Calculus

Input: a clause set $\mathcal{S}$

Output: "unsatisfiable" or "satisfiable" (if terminates)

Note: Strategy much like in inner loop of propositional DPLL:



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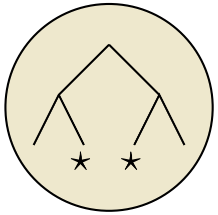

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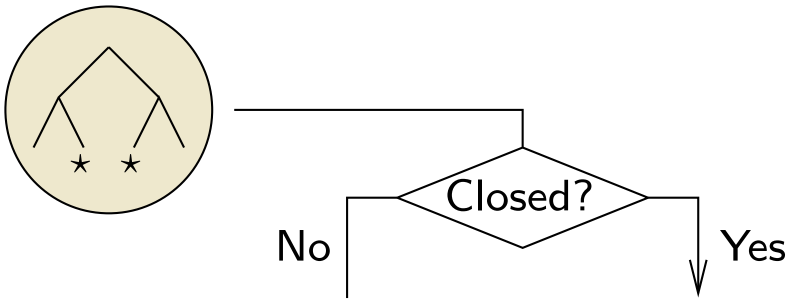

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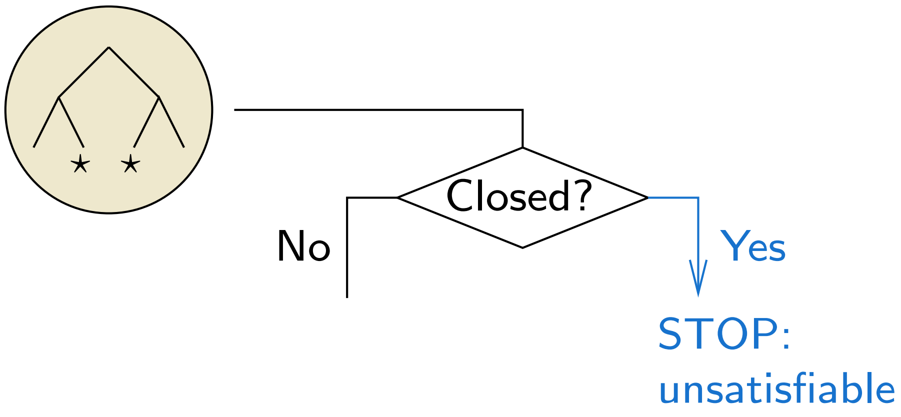

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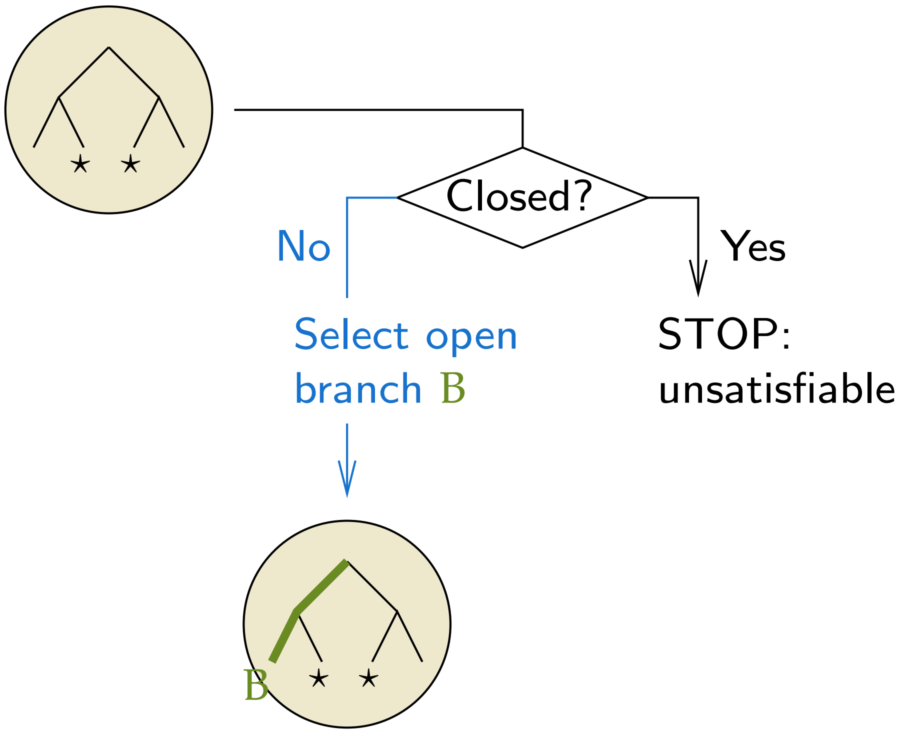

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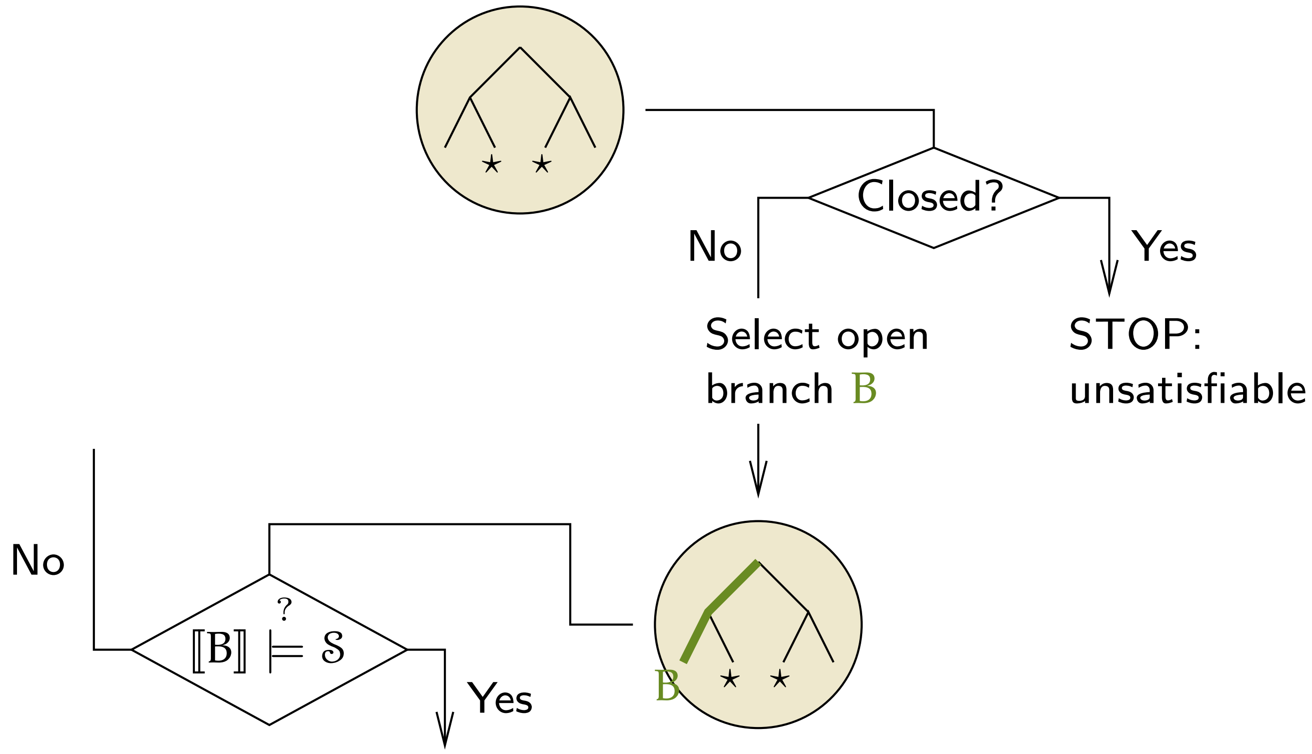

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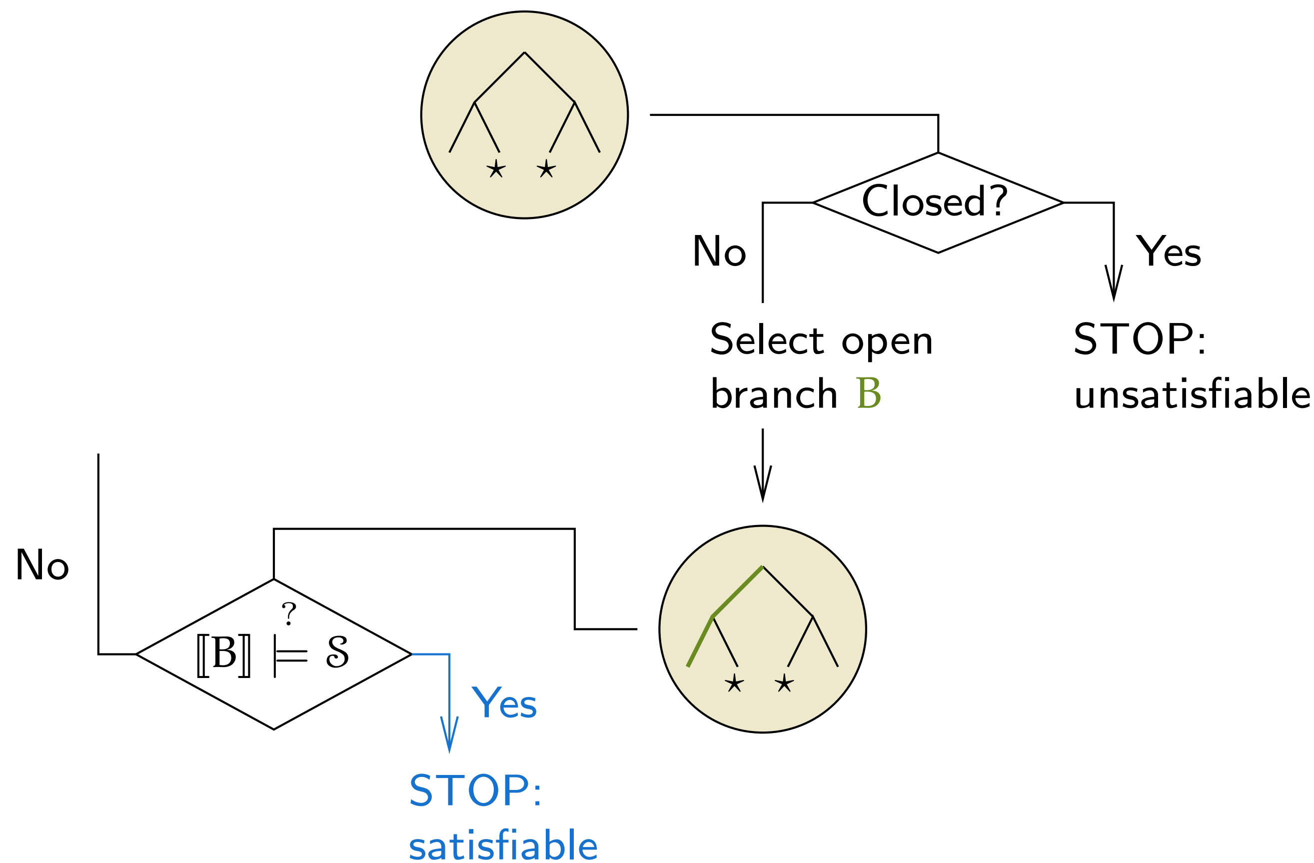

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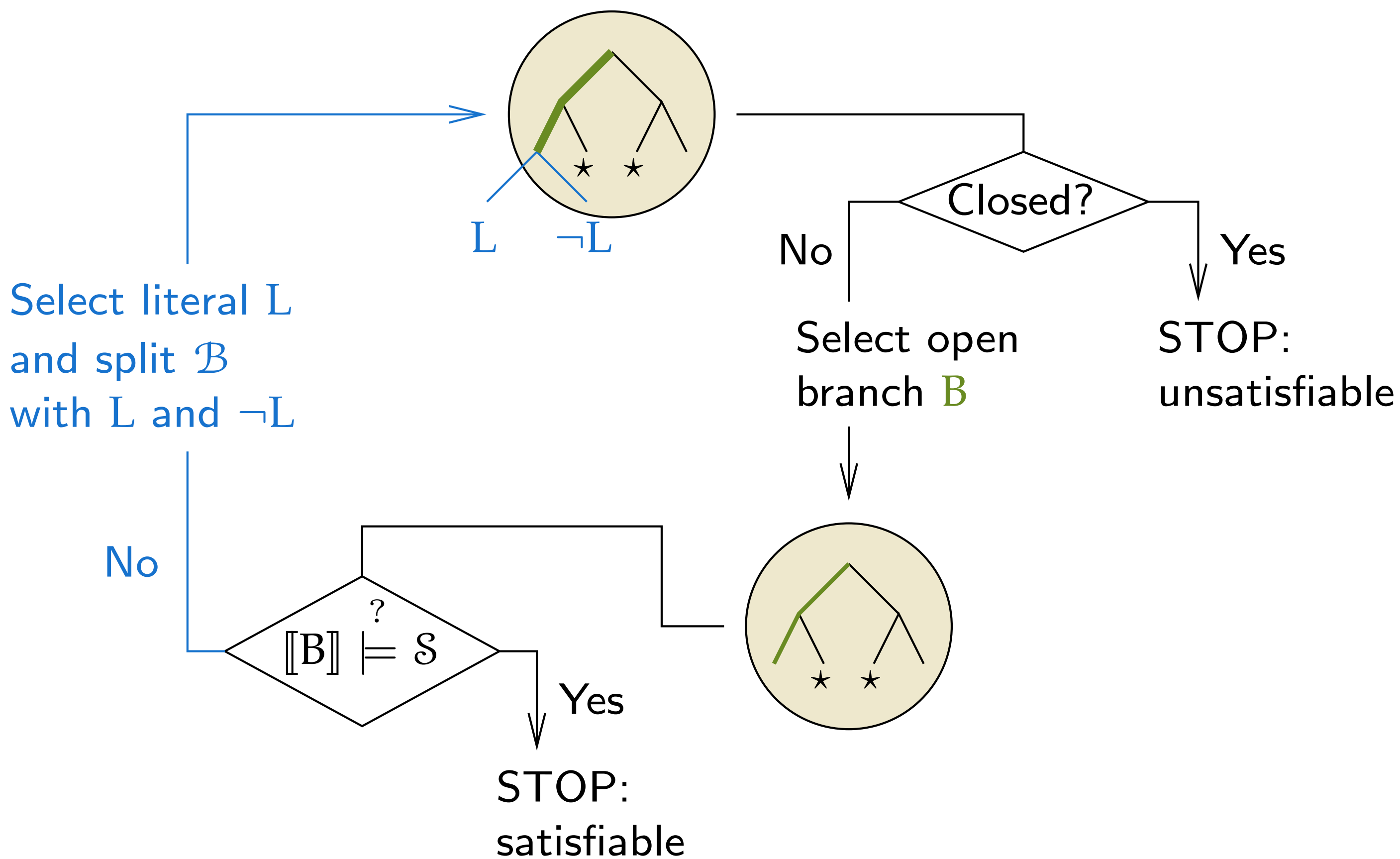

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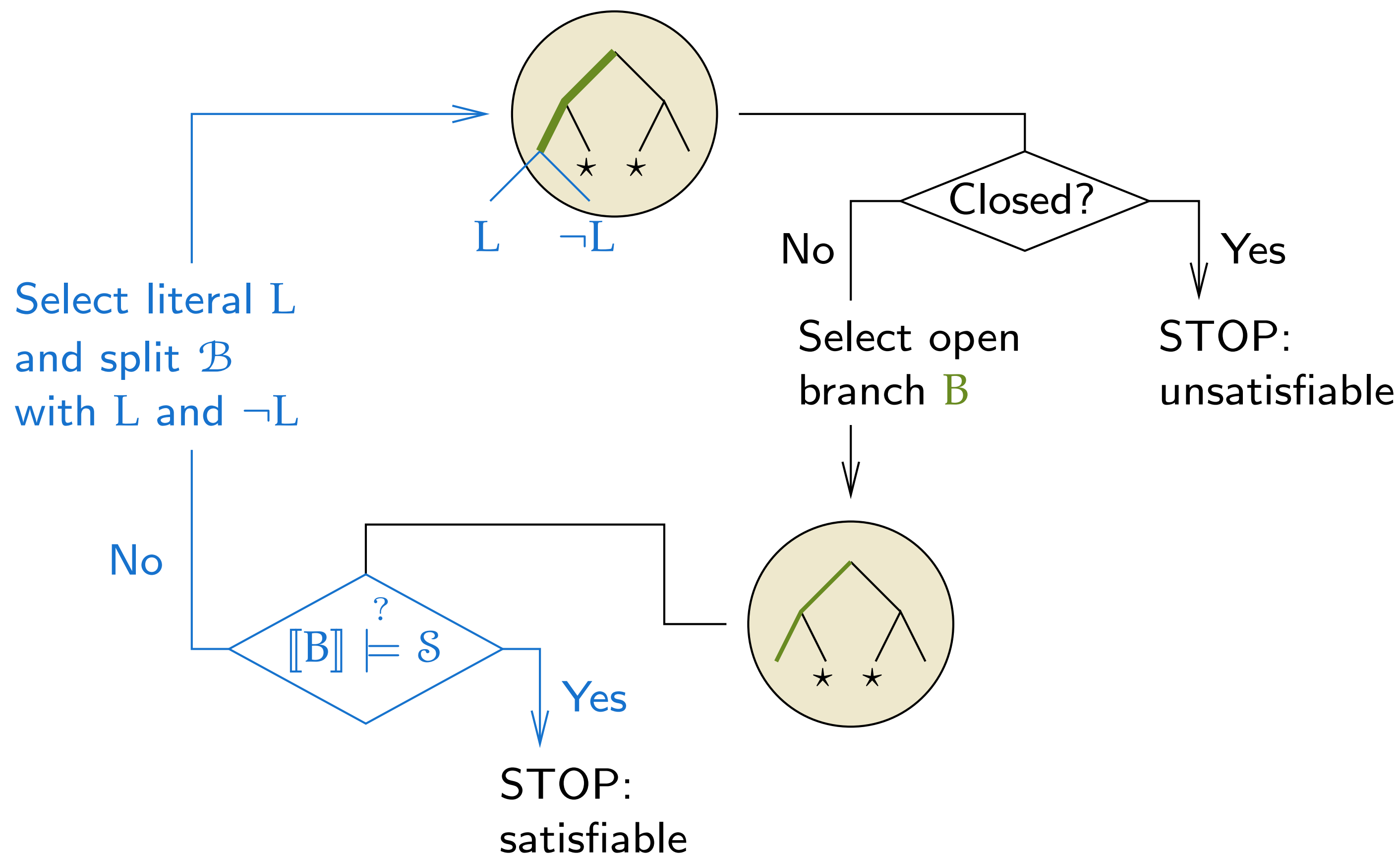
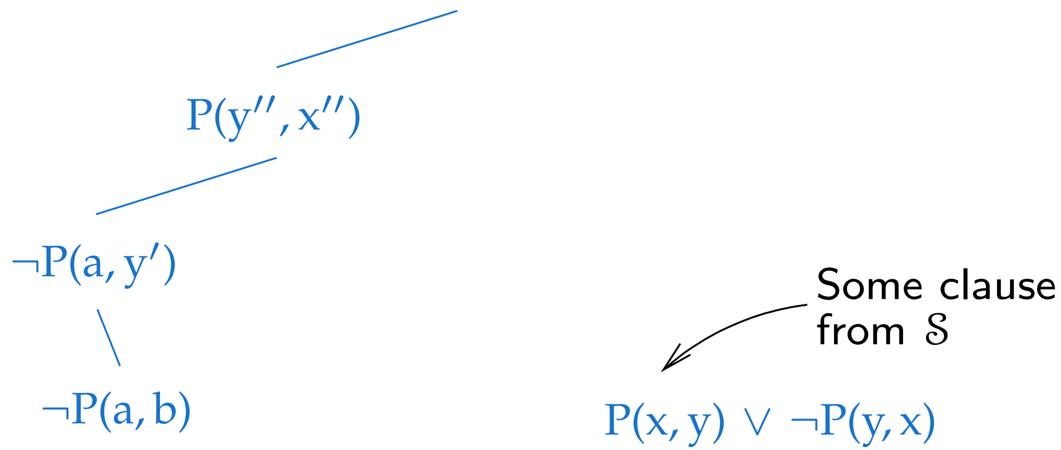


Next: Testing $[\![B]\!] \models \mathcal{S}$ and splitting

Calculus: The Splitting Rule

Purpose: Satisfy a clause that is currently "false"



1.

2.

3.

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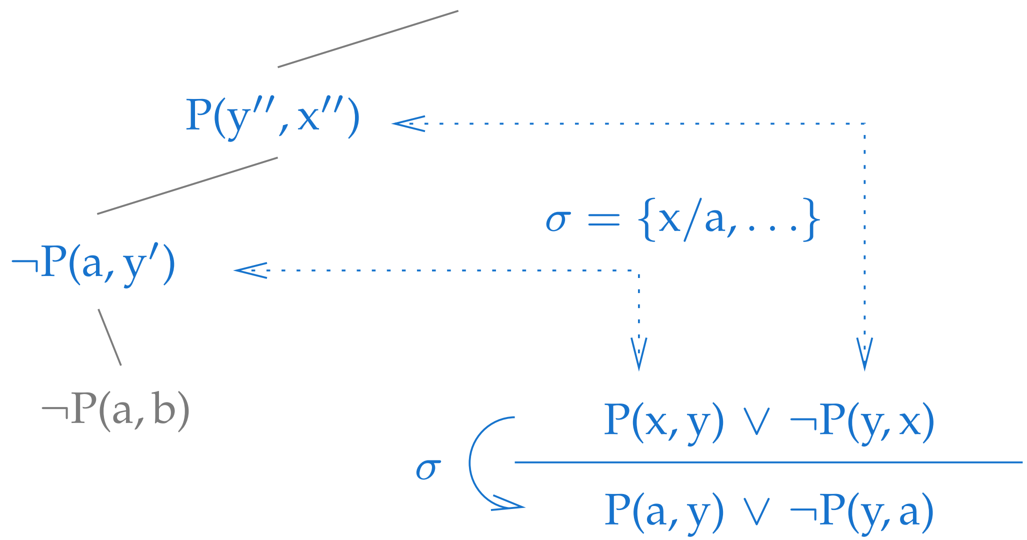

1. Compute simultaneous most general unifier σ
2.
3.

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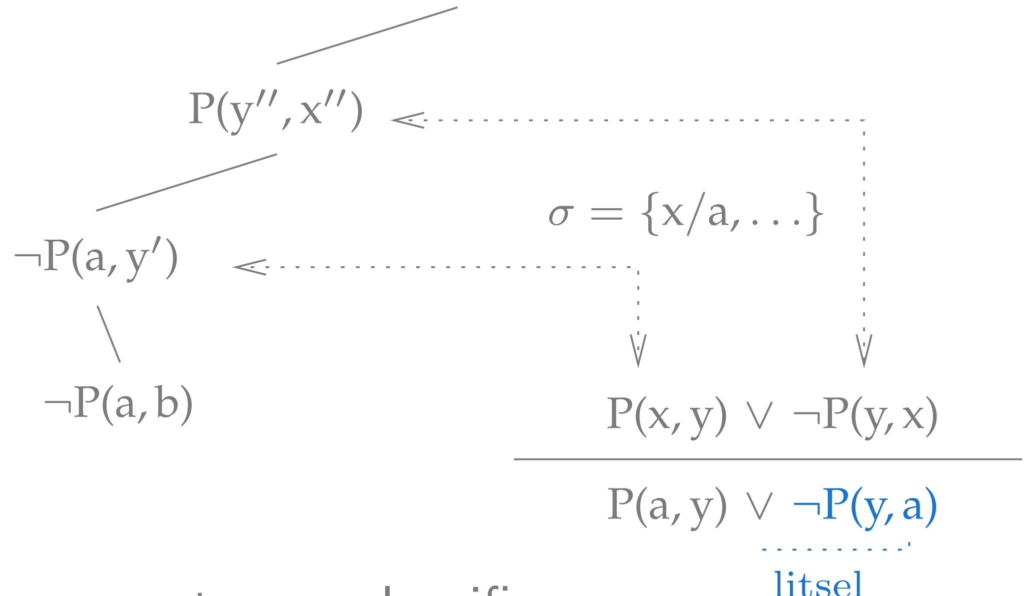

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2. Select from clause instance a literal not on branch
3.

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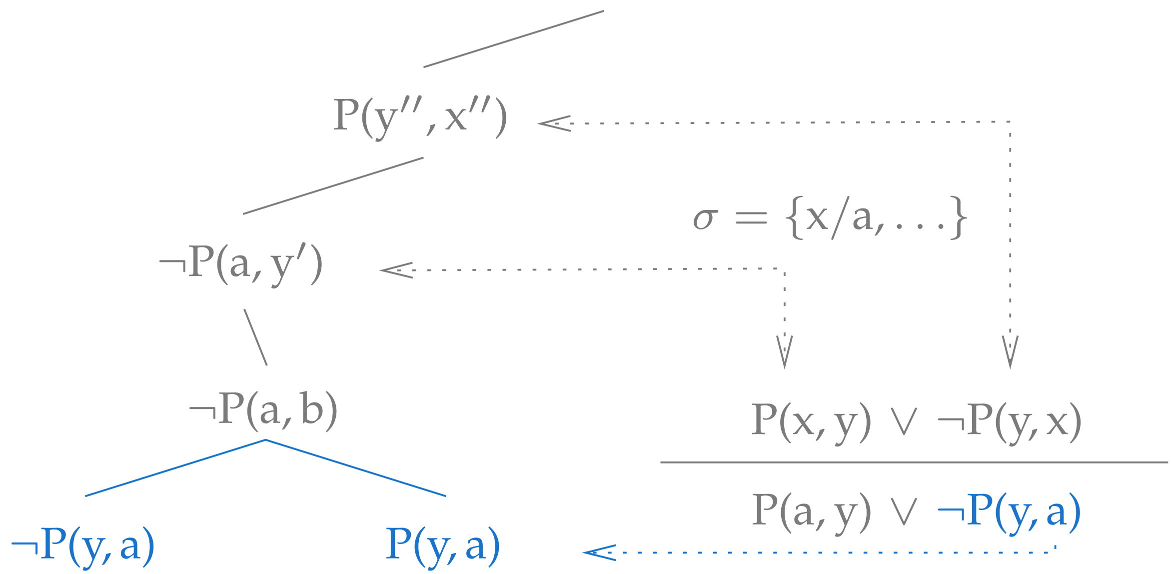

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Calculus: The Splitting Rule

Purpose: Satisfy a clause that is currently “false”

$P(y'', x'')$

$\neg P(a, y')$

$\neg P(a, b)$ $\qquad\qquad$ $P(x, y) \vee \neg P(y, x)$

$$\{\neg P(a, c), P(c, a), \ldots\} \quad \not\models \quad P(a, c) \vee \neg P(c, a)$$

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This split was really necessary!

Calculus: The Splitting Rule

Purpose: Satisfy a clause that is currently "false"

$$P(y'', x'')$$

$$\neg P(a, y')$$

$$\neg P(a, b) \qquad\qquad P(x, y) \lor \neg P(y, x)$$

$$\{\neg P(a, c), P(c, a), \ldots\} \not\models P(a, c) \lor \neg P(c, a)$$

1. Compute simultaneous most general unifier σ
2. Select from clause instance a literal not on branch
3. Split with this literal

This split was really necessary!

Proposition: If $[\![\mathcal{B}]\!] \not\models \mathcal{S}$, then split is applicable to some clause from $\mathcal{S}$

Calculus: The Splitting Rule – Another Example

Purpose: Satisfy a clause that is currently "false"

$\mathrm{P}(y'', x'')$

$\neg \mathrm{P}(a, y')$

$\neg \mathrm{P}(a, b)$

Some clause from $\mathcal{S}$

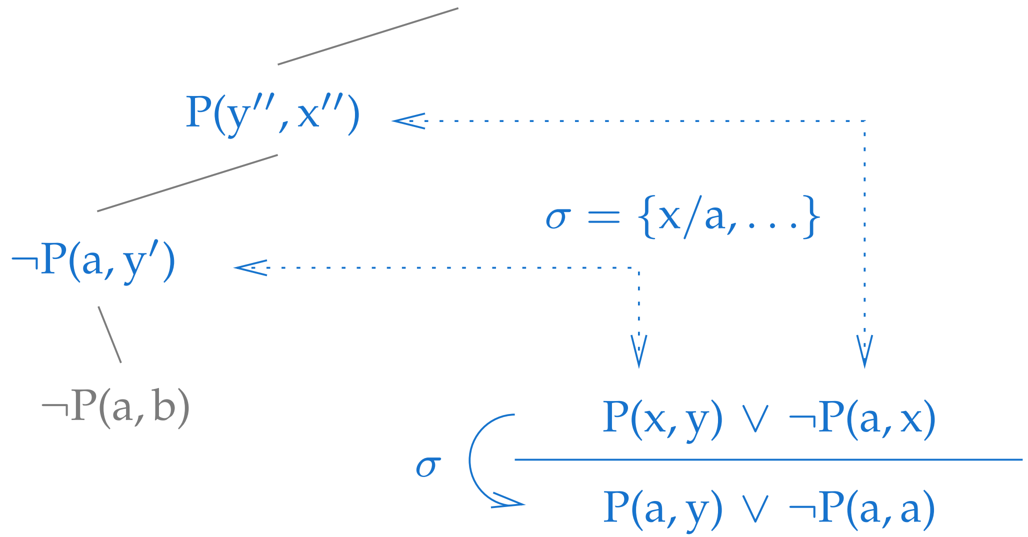
$\mathrm{P}(x, y) \vee \neg \mathrm{P}(a, x)$

1.

2.

Calculus: The Splitting Rule – Another Example

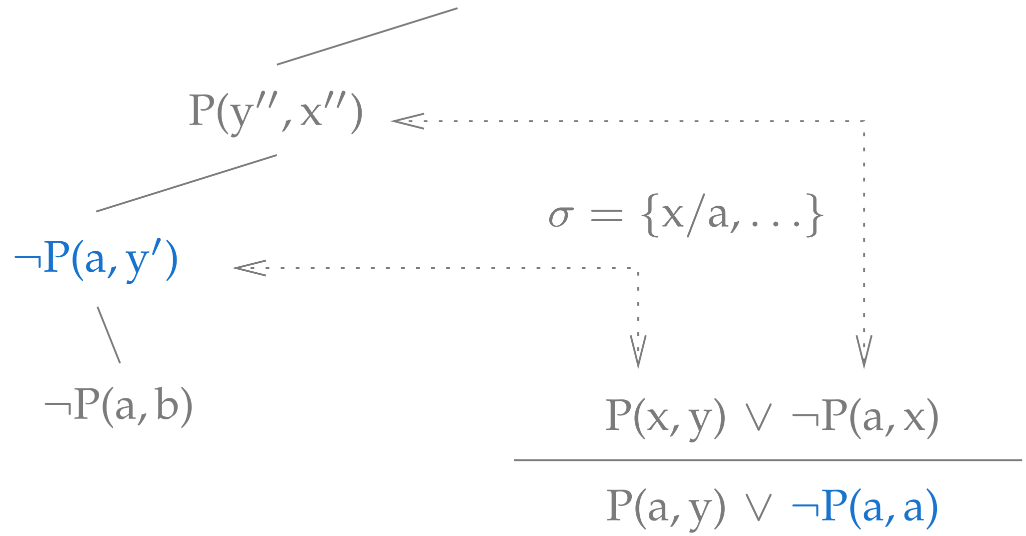
Purpose: Satisfy a clause that is currently "false"



1. Compute MGU σ of clause against branch literals
2.

Calculus: The Splitting Rule – Another Example

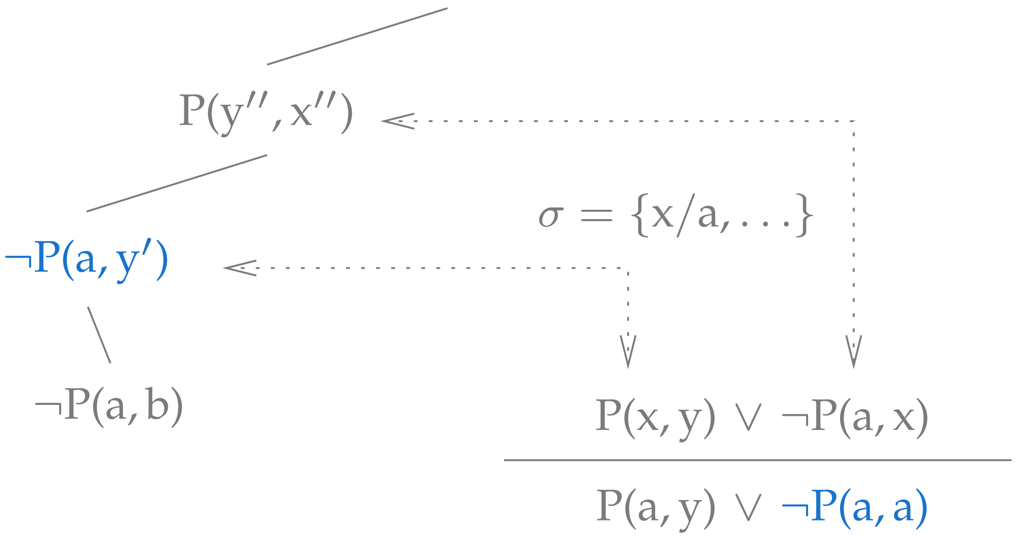
Purpose: Satisfy a clause that is currently “false”



1. Compute MGU σ of clause against branch literals
2. If clause contains “true” literal, then split is not applicable

Calculus: The Splitting Rule – Another Example

Purpose: Satisfy a clause that is currently “false”



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Non-applicability is a redundancy test

Calculus: The Splitting Rule – Another Example

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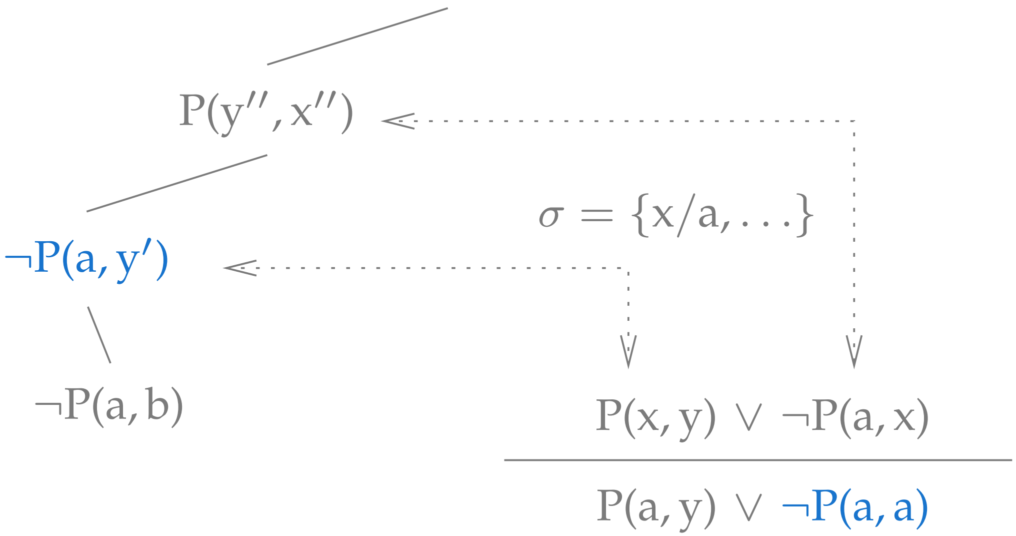


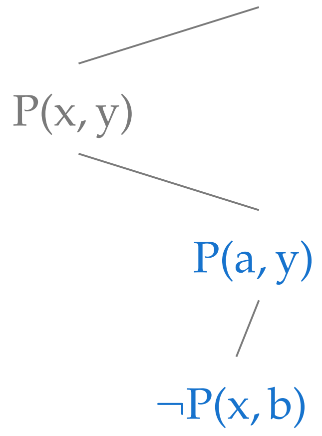
1. Compute MGU σ of clause against branch literals
2. If clause contains "true" literal, then split is not applicable

Non-applicability is a redundancy test

Proposition: If for no clause split is applicable, $[\![\mathcal{B}]\!] \models \mathcal{S}$ holds

Calculus: The Commit Rule

Purpose: Achieve consistency of interpretation associated to branch



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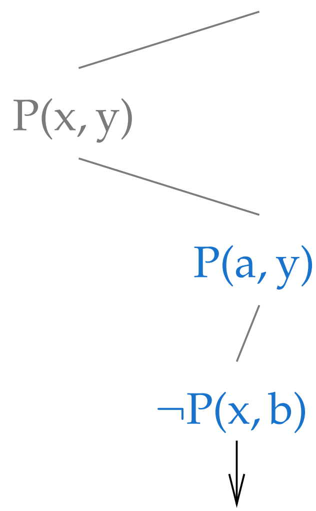

Problem: $\{P(a,b), \neg P(a,b)\}$ is inconsistent!

Calculus: The Commit Rule

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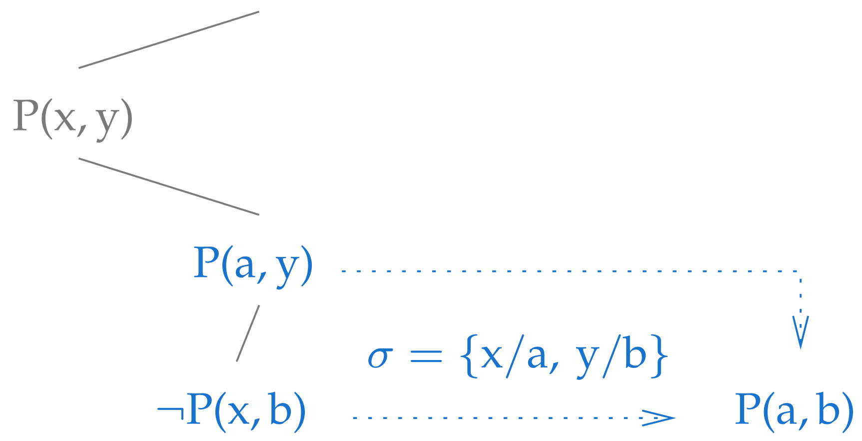

1. Compute a MGU σ of branch literals with opposite sign
2.

Calculus: The Commit Rule

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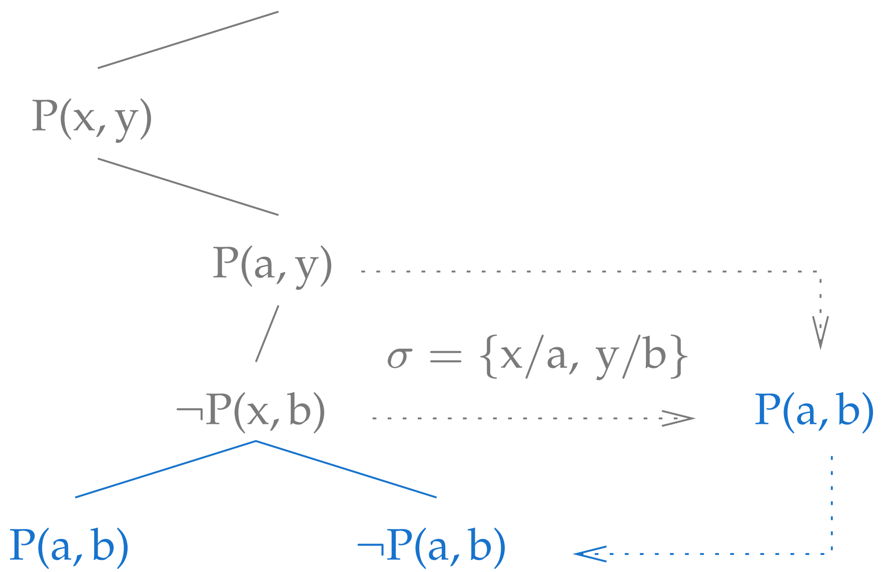

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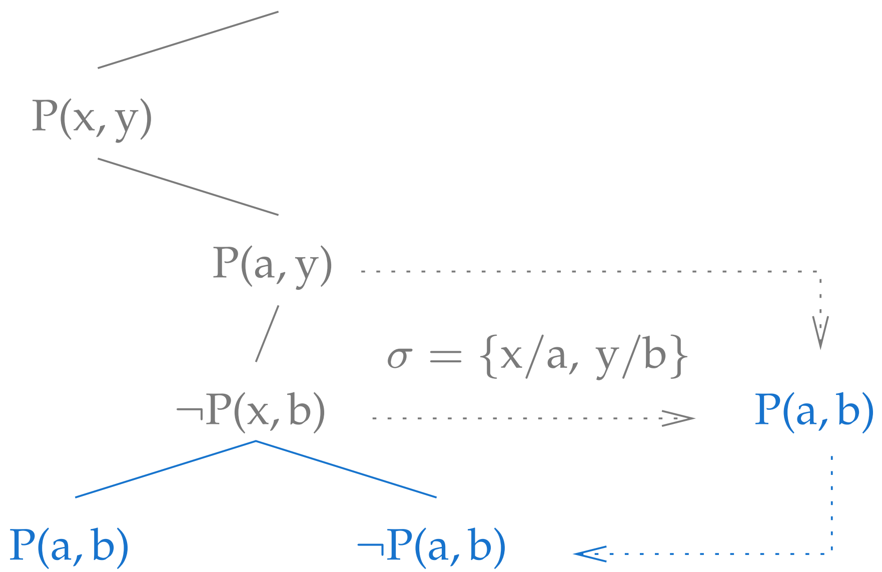

1. Compute a MGU σ of branch literals with opposite sign
2. Split with instance, if not on branch

Now have removed the inconsistency

FDPLL Complete Example

```
(1)  train(X,Y) ; flight(X,Y).          %% train from X to Y or flight from X to Y.
(2)  -flight(koblenz,X).                %% no flight from koblenz to anywhere.
(3)  flight(X,Y) :- flight(Y,X).        %% flight is symmetric.
(4)  connect(X,Y) :- flight(X,Y).       %% a flight is a connection.
(5)  connect(X,Y) :- train(X,Y).        %% a train is a connection.
(6)  connect(X,Z) :- connect(X,Y),      %% connection is a transitive relation.
                     connect(Y,Z).
```

FDPLL Complete Example

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```

Computed Model (as output by implementation)

```
+ flight(X, Y)
- flight(koblenz, X)
- flight(X, koblenz)
+ train(koblenz, Y)
+ train(Y, koblenz)
+ connect(X, Y)
```

FDPLL Model Computation Example - Derivation

$\langle$empyty tree$\rangle$

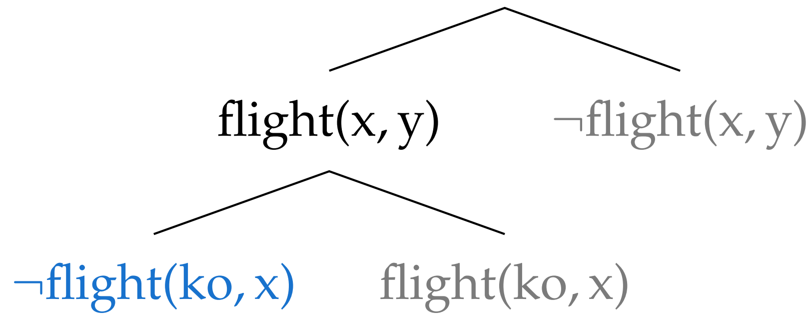
Clause instance used in inference: $\mathrm{train}(x, y) \vee \mathrm{flight}(x, y)$

FDPLL Model Computation Example - Derivation

$\mathrm{flight}(x, y) \qquad \neg\mathrm{flight}(x, y)$

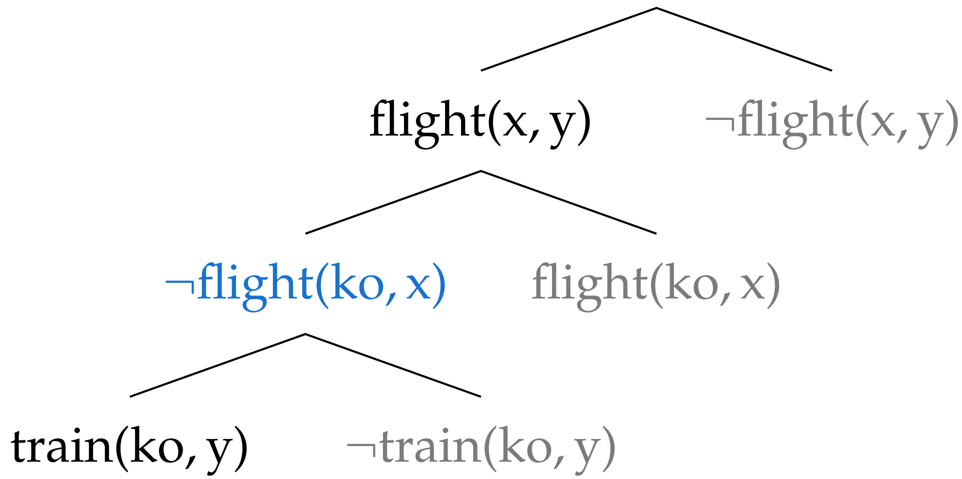
Clause instance used in inference: $\neg\mathrm{flight}(ko, x)$

FDPLL Model Computation Example - Derivation



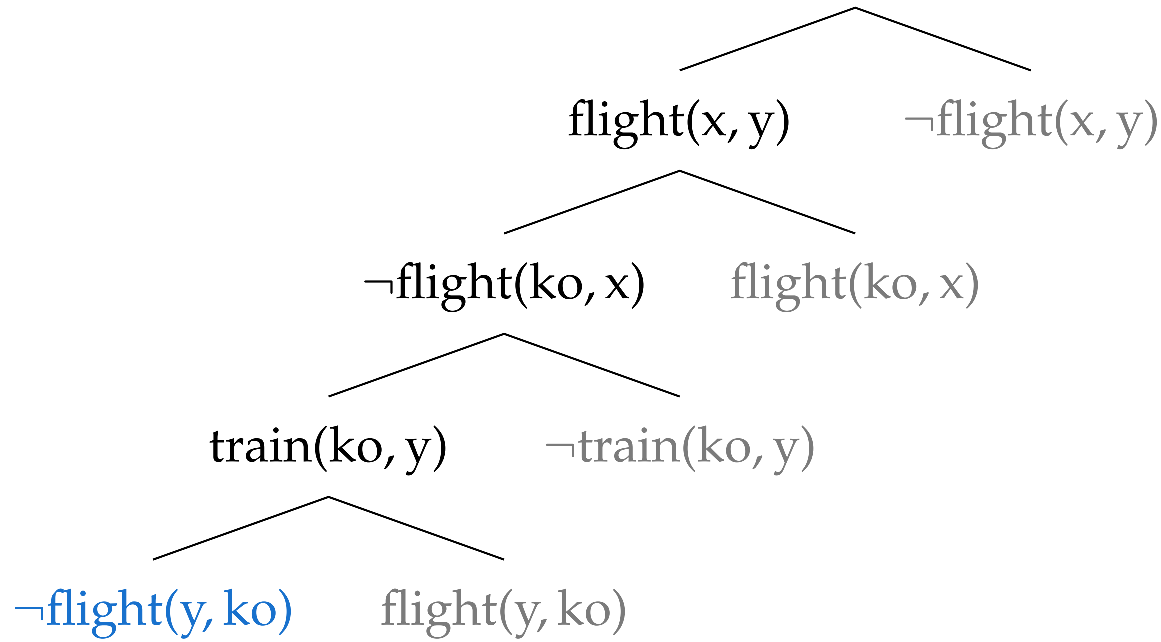
Clause instance used in inference: $\mathrm{train}(\mathrm{ko}, y) \vee \mathrm{flight}(\mathrm{ko}, y)$

FDPLL Model Computation Example - Derivation



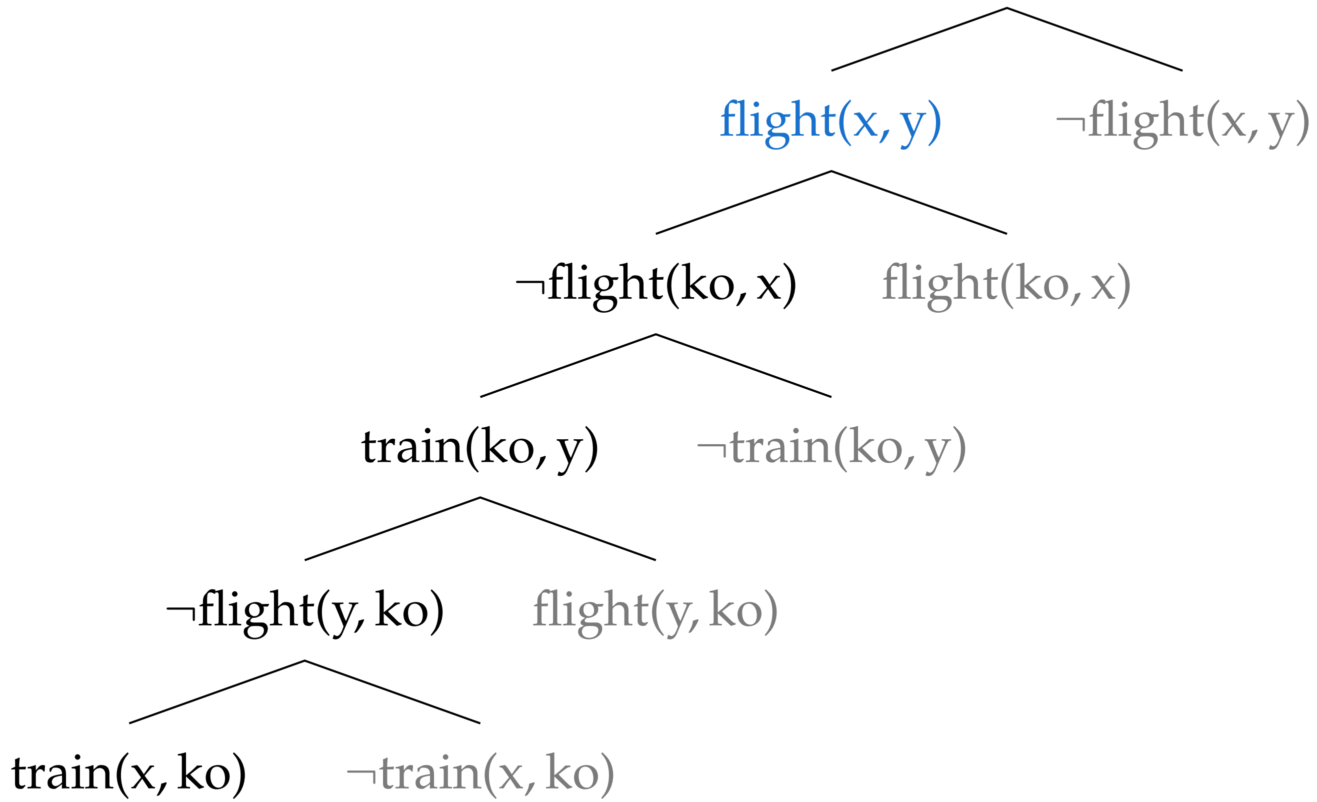
Clause instance used in inference: $\mathrm{flight}(ko, y) \lor \neg\mathrm{flight}(y, ko)$

FDPLL Model Computation Example - Derivation



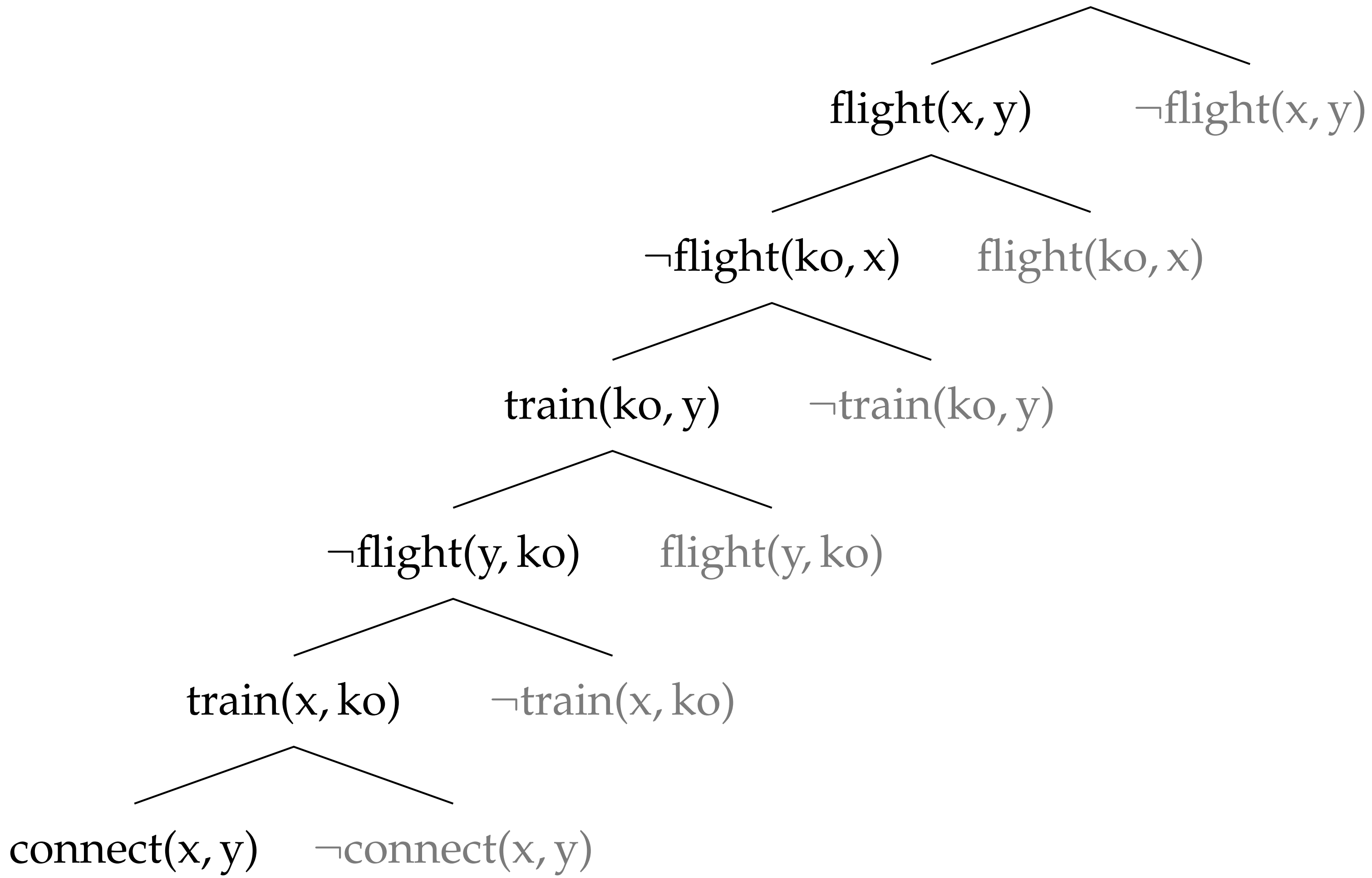
Clause instance used in inference: $\mathrm{train}(x, ko) \lor \mathrm{flight}(x, ko)$

FDPLL Model Computation Example - Derivation



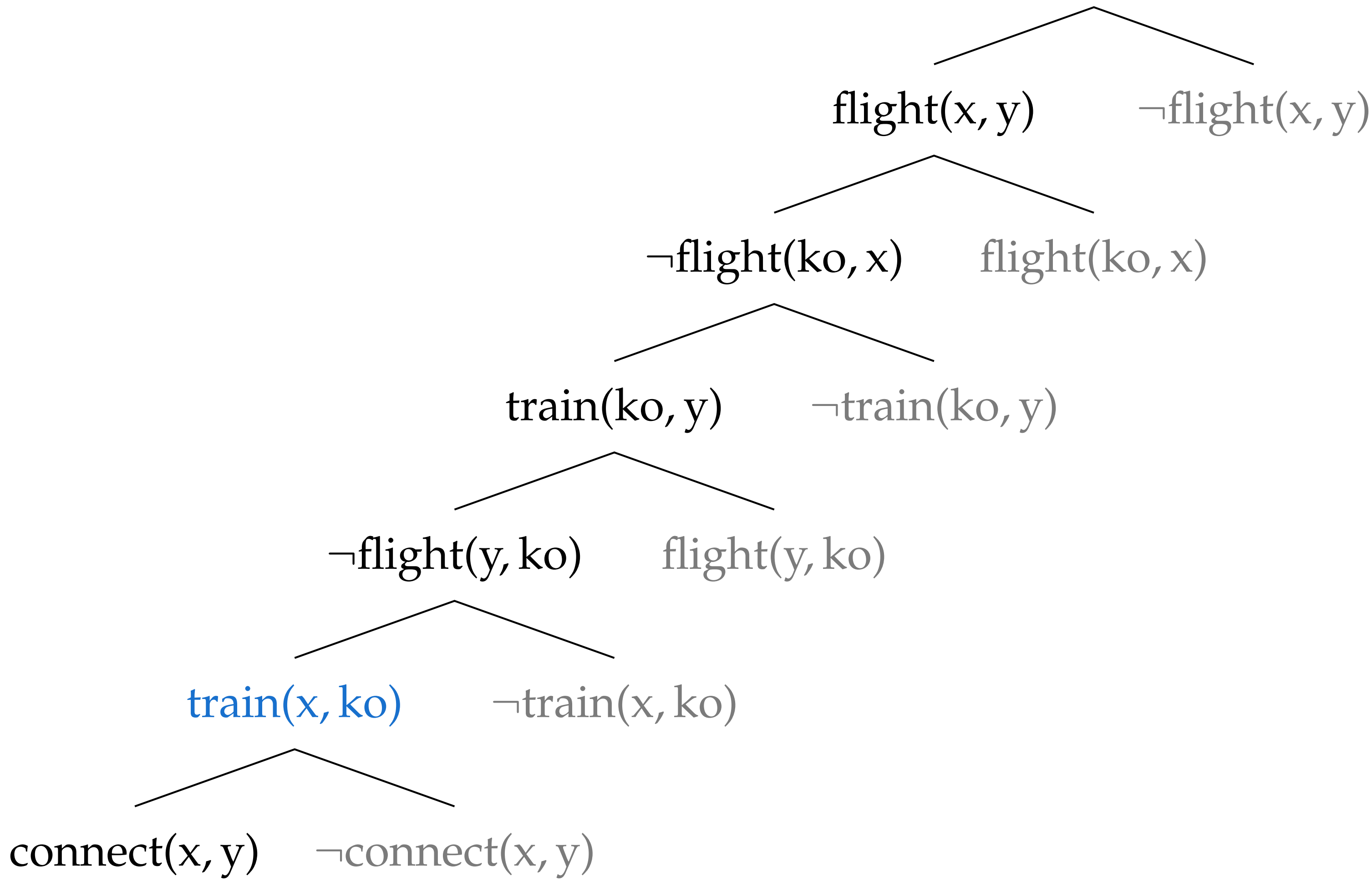
Clause instance used in inference: $\mathrm{connect}(x, y) \vee \neg\mathrm{flight}(x, y).$

FDPLL Model Computation Example - Derivation



Done. Return "satisfiable with model $\{\mathrm{flight}(x, y), \ldots, \mathrm{connect}(x, y)\}$"

FDPLL Model Computation Example - Derivation



Done. Return "satisfiable with model $\{\mathrm{flight}(x, y), \ldots, \mathrm{connect}(x, y)\}$"

Redundancy: Instance not used in inference: $\mathrm{connect}(x, ko) \vee \neg\mathrm{train}(x, ko)$

Optional Inference Rule – Universal Splits

(1) $\mathrm{P(x)}$ (2) $\neg\mathrm{P(x)} \vee \mathrm{Q(x)}$

Split based on tautology $\forall \mathrm{x}\ \mathrm{P(x)} \vee \neg\forall \mathrm{x}\ \mathrm{P(x)}$:

$\forall \mathrm{x}\ \mathrm{P(x)}$ $\qquad$ $\underset{\star}{\neg\mathrm{P(sk1)}}$

Sources for Universal Splits

- Unit input clauses

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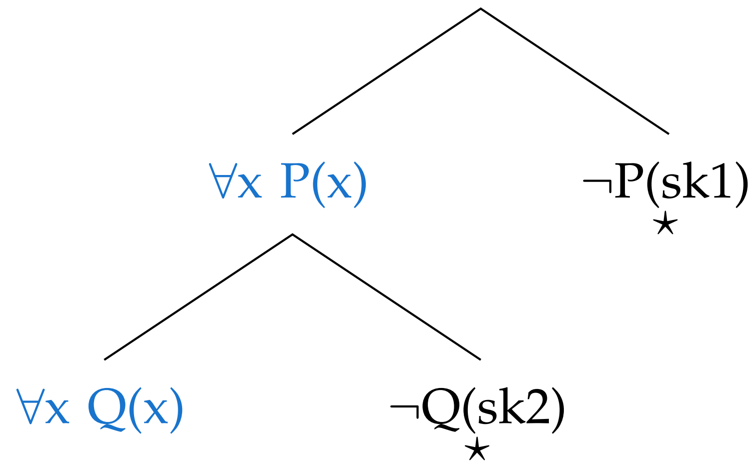

Sources for Universal Splits

- Unit input clauses
- Resolving away $n - 1$ literals from an n-literal clause (UR-Resolution)

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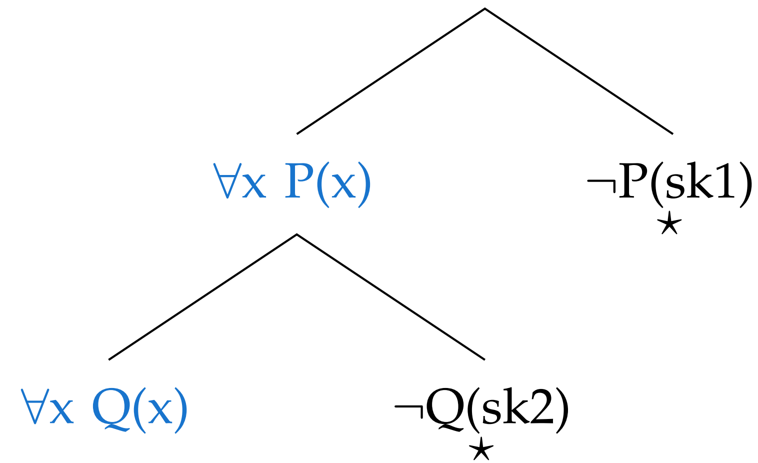


Sources for Universal Splits

- Unit input clauses
- Resolving away $n-1$ literals from an n-literal clause (UR-Resolution)

Advantages: – No “exceptions” permitted, hence much better efficiency
– Subsumption

Calculus: Summary / Properties

Summary

- DPLL data structure lifted to first-order logic level
- Two simple inference rules, controlled by unification
- Computes with interpretations/models
- Semantical redundancy criterion

Calculus: Summary / Properties

Summary

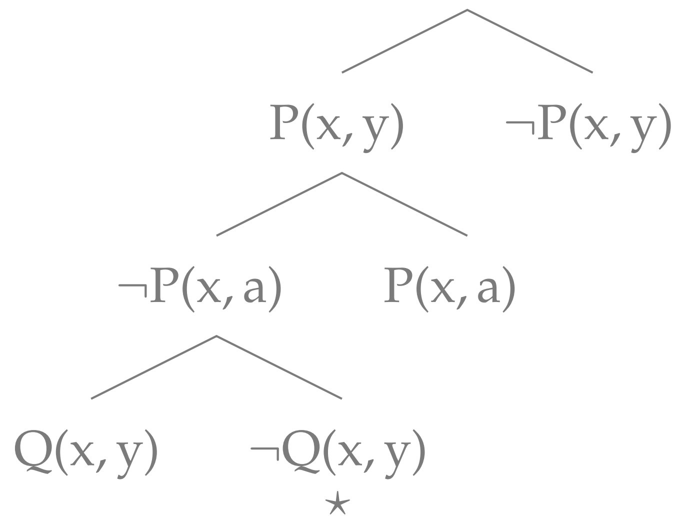
- DPLL data structure lifted to first-order logic level
- Two simple inference rules, controlled by unification
- Computes with interpretations/models
- Semantical redundancy criterion

Properties

- Soundness and completeness (with fair strategy).
- Extension: More efficient reasoning with unit clauses (e.g. $\forall x\ P(x, a)$)
- Proof convergence (avoids backtracking the semantics trees)
- Decides function-free clause logic (Bernays-Schönfinkel class)
  Covers e.g. Basic modal logic, Description logic, DataLog
  Returns model in satisfiable case
  But: Resolution better on other classes!

[Fermüller et. al. Handbook AR 2001 (e.g. Gödel class, Monadic class, Guarded Fragment,...)]

First-Order Semantic Trees



Issues:

- How are variables treated?
  (a) Universal, as in Resolution?, (b) Rigid, as in Tableaux? (c) Schema!
- How to extract an interpretation from a branch? ✔
- When is a branch closed? ✔
- How to construct such trees (calculus)? ✔

Overview

Propositional DPLL as a semantic tree method ✔

First-Order DPLL so far ✔

FDPLL ✔

Relation to other calculi

Families of First-Order Logic Calculi

Consider a transitivity clause $\mathrm{P}(x,z) \leftarrow \mathrm{P}(x,y) \wedge \mathrm{P}(y,z)$.

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Resolution:

$$\begin{aligned}
\mathrm{P}(x,z') &\leftarrow \mathrm{P}(x,y) \wedge \mathrm{P}(y,z) \wedge \mathrm{P}(z,z') \\
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Rigid Variable Approaches:

FO-DPLL: [Chang&Lee 73]
Tableaux and CM: [Peltier, IGPL 99], [Baumgartner et al, CADE 99], [Beckert, FTP 2000], [Giese, CADE 01]

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- Unpredictable number of variants, weak redundancy test
- Difficult to avoid unnecessary (!) backtracking
- Difficult to extract model

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FDPLL in this tradition, but

- need not keep instances
- conceptually different: – binary splitting
  – based on first-order interpretations

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Conclusions

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- In Eclipse Prolog, $\approx$ 1300 LoC, `http://www.uni-koblenz.de/~peter/FDPLL/`

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- Success rates:
  State-of-the-art systems: $\approx$ 55%, FDPLL: $\approx$ 40%
  State-of-the-art Resolution systems, 70's technology: $\approx$ 30%

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- Serious implementation (OCaml)